Volume. 7 No 1 June (2024)

e-ISSN: **2615-3270**
p-ISSN: **2615-3599**

Eigen Mathematics Journal

Homepage jurnal: https://eigen.unram.ac.id/index.php/eigen

Natural Cubic Spline Method as a Method in Constructing a Life Table in Gegelang Village West Lombok

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A B S T R A C T

This research aims to reconstruct a life table based on real data obtained in Gegelang Village, West Lombok. The data used in this research is the population in 2016, the death rate in 2014-2018 and the birth rate in 2014-2018. The first step taken was to compile a rough life table using the partial data situation and full data situation methods. Both methods are included in the maximum likelihood method. After carrying out calculations, different life expectancy figures are obtained. The respective calculation results were 62.21 years for the partial data situation method and 73.07 years for the full data situation method. Next, a graduation is carried out using the natural cubic spline method on the life table obtained from a rough life table model calculation. The graphic model produced by the rough life table is fluctuating so it is necessary to graduate using the natural cubic spline method to obtain a monotonically decreasing graph. The life table model chosen for graduation is a life table whose life expectancy is close to the life expectancy of West Lombok Regency in 2015, namely 65.1 years. After graduation, the new life expectancy was found to be 66.92 years.

Keywords: Life Table, Life Expectancy Rate, Graduation

Submitted: 22-05-2024; Accepted: 29-06-2024 Doi: https://doi.org/10.29303/emj.v7i1.215

1. Introduction

Death is the permanent loss of all signs of life that occurs after birth. The death rate shows a probability value that individuals who have reached a certain age will die within one year. Various factors influence the high and low mortality rates, for example age structure, gender, type of work, socio-economic status, environmental conditions and so on. Information about deaths is very important for the government to be able to reflect development conditions (Wirosuhardjo et al, 1985).

One of the ways we can see the high or low death rate of the population of an area is the hope rate in that area. Life expectancy is one of the indicators used to assess the level of people's welfare and one of the tools used to evaluate government performance (Muda et al, 2019). The greater the life expectancy of a population, the degree of welfare of the population will also increase. Life expectancy is one of the benchmarks that illustrates the success of government programs to improve people's welfare, especially in the health sector (Ginting & Lubis, 2023). So it is very important to carry out further studies on what methods are appropriate for producing life expectancy (BPS, 2005). Facts show that the life expectancy of developed countries is different from developing countries. We can see this from the socio-economic conditions of a country which greatly influence the size of the life expectancy. To see the life expectancy of a country, one can look at the life table of that country.

Life table is a table that describes the chances of an individual surviving from the year of birth to a certain age (Brown, 1997). Life tables are a way to analyze death rates for certain ages, calculating the probability of survival and the average life expectancy of the population. There are two forms of life tables, including: complete life table (complete life table) and abridged life table (short life table). A complete life table is a form of life table that is compiled at one year intervals. Meanwhile, a concise life table is a form of life table

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that is compiled at intervals of more than one year, usually five or ten years.

Until now, our country still does not have a life table compiled based on death data according to age which can be used primarily to calculate life expectancy. The life table that is still used by Indonesia is an approach adapted to the Western Model Coale-Demeny Life Table. Fauzi and friends (2023) conducted research by calculating life expectancy in Gegelang Village, West Lombok. In their research they carried out a graduation process using the Quadratic Spline and Heligman Pollard methods. After carrying out the graduation process, the best method was chosen based on the value of life expectancy which was closest to the life expectancy of West Lombok Regency in 2015. From the results of the calculations, the Heligman Pollard method was chosen as the method that matched the existing data (Fauzi, 2023). Based on the problems above, in this research a life table will be prepared based on real data in Gegelang Village, West Lombok using the graduation method is natural cubic spline after previously calculating the probability of incomplete death data using the maximum likelihood method.

2. Methodology

In this research, several stages were carried out. The first stage was data collection in Gegelang Village, West Lombok Regency, West Nusa Tenggara. The sample is part of a population that has the characteristics of the population itself (Amin et al, 2023). Meanwhile, population is all research objects that have certain characteristics (Dewi & Pardede, 2021). The sample data used in this research is secondary data obtained from population data in Gegelang Village, Lingsar District, West Lombok Regency, West Nusa Tenggara. The data used is the population in 2016, the death rate for the year 2014-2018 and the birth rate for the year 2014-2018. Next, the probability parameter q (estimator of death) is calculated using the maximum likelihood estimator to obtain a rough life table model. Those included in the maximum likelihood estimator category are partial data situations and full data situations. The next step is to graduate the life table obtained from the previous step using the graduation method until the appropriate graduation method is obtained. The graduation method used in this research is Natural Cubic Spline. After graduation, a new life expectancy number will be obtained.

2.1. Calculating the Probability of Death for Incomplete Data Samples Using the Maximum Likelihood Estimator Method

The method used in this research for incomplete data samples is the Maximum Likelihood Estimator method or often called the Maximum Likelihood method. Using this method on the complete data sample, two methods will be used to estimate the probability of death based on data obtained from Gegelang Village, West Lombok. The two methods are the Partial Data Situation method and the Full Data Situation method.

In the partial data situation, what is known in the age interval $(x, x+5]$ is only the number of individual deaths from the data obtained. Because what is known is only the number of individual deaths in the age interval $(x, x+5]$, then the parameter estimate for the probability of death is assumed to follow a binomial distribution so that the formula is used to calculate the probability of death in this method, namely

$$\hat{q}_x^P = \frac{\sum_{i=1}^{N_x} \delta_i}{N_x} = \frac{D_x}{N_x}. \tag{1}$$

by :
x = individual
$\hat{q}_x^P$ = the probability of death is calculated using the partial data situation method
N_x = the number of individuals observed
D_x = individual death

The full data situation is used if from the data obtained it is known the exact age of death of the ith individual in the age interval $(x, x+5]$ for a five year interval. Suppose there are N_x individuals observed in the age interval $(x, x+5]$ with different exact death values. In this method, the death contribution of the ith individual is a probability density function, where the likelihood function is assumed to be the product of the age interval $(x, x+5]$. So by using Eq

$$f_x(t) = \frac{f_0(x+t)}{S_0(x)} = \frac{S_0(x+t)}{S_0(x)}\lambda_0(x+t) = {}_tp_x \cdot \mu_{x+t}$$

The probability density function is obtained as follows

$$L_i = {}_{t_i - r_i}p_{x+r_i} \cdot \mu_{x+t_i} \tag{2}$$

where $0 \leq r_i < t_i \leq 5$ with $t_i = s_i$ for $\delta_i = 0$ and $t_i = \iota_i$ for $\delta_i = 5$. Next, using the equation $L(\theta; x) = \prod_{i=1}^{N} f(x_i; \theta), \theta \in \Omega$ the likelihood function is obtained as follows

$$L = \prod_{i=1}^{N_x} {}_{t_i - r_i}p_{x+r_i} \cdot \mu_{x+t_i}. \tag{3}$$

Let l_{x+t} in the equation $l_{x+t} = a + bt$ is assumed to be exponential, so we get

$$l_{x+t} = a \cdot b^t \tag{4}$$

If let $t = 0$ so $l_x = a$, $t = 1$ so a $l_{x+1} = a \cdot b$ which mean $b = \frac{l_{x+1}}{a} = \frac{l_{x+1}}{l_x} = p_x$. So the equation (4) can be formulated in the form

$$l_{x+t} = l_x \cdot (p_x)^t \tag{5}$$

By using the equation (5) for $0 \leq t \leq 1$ is obtained, namely

$${}_tp_x = \frac{l_{x+t}}{l_x} = \frac{l_x \cdot (p_x)^t}{l_x} = (p_x)^t \tag{6}$$

And

$$ {}_tq_x = 1 - {}_tp_x = 1 - (p_x)^t = 1 - (1 - q_x)^t \quad (7)$$

In addition, the formula for the probability of survival and the probability of death in the subinterval $[x + a, x + b]$ where $0 \leq a < b \leq 1$ is

$$ {}_{b-a}p_{x+a} = \frac{l_{x+b}}{l_{x+a}} = (p_x)^{b-a} \quad (8)$$

And

$$\begin{aligned} {}_{b-a}q_{x+a} &= \frac{{}_{a|b-a}q_x}{{}_ap_x} \\ &= \frac{{}_ap_x - {}_bp_x}{{}_ap_x} \\ &= \frac{(p_x)^a - (p_x)^b}{(p_x)^a} \\ &= 1 - (p_x)^{b-a} \end{aligned} \quad (9)$$

Next, the FOM function is defined as

$$\mu_{x+t} = \frac{-\frac{d}{dt}l_{x+t}}{l_{x+t}} = \frac{-l_x(p_x)^t \cdot \ln(p_x)}{l_x(p_x)^t} = -\ln(p_x) = \mu^* \quad (10)$$

So based on equation (10), we get

$$\begin{aligned} -\ln(p_x) &= \mu^* \\ \ln(p_x) &= -\mu^* \\ p_x &= \exp(-\mu^*) \end{aligned} \quad (11)$$

So that equations (8) and (9) can also be formulated as

$$ {}_{b-a}p_{x+a} = \exp\big(-\mu^*(b - a)\big) \quad (12)$$

And

$$ {}_{b-a}q_{x+a} = 1 - \exp\big(-\mu^*(b - a)\big) \quad (13)$$

Based on equation (13), if it is assumed that a = beginning of observation $((r_i)$, b = end of observation (t_i) for the ith individual, then equation (13) can also be formulated as

$$ {}_{t_i-r_i}p_{x+r_i} = \exp\big(-\mu(t_i - r_i)\big) \quad (14)$$

where the value μ is constant.

Next, using equations (2), (17) and (18), we obtain the parameter estimator μ for each age interval $(x, x + 5]$ namely

$$L(\mu) = \prod^{N_x} \big(\exp\big(-\mu(t_i - r_i)\big)\big) \cdot (\mu)^{D_x} \quad (15)$$

$$l'(\mu) = \frac{D_x}{\mu} - \sum_{i=1}^{N_x}(t_i - r_i) = 0$$

$$\hat{\mu}_x = \frac{D_x}{\sum_{i=1}^{N_x}(t_i - r_i)}$$

so that based on equation (10), the formula for estimating the probability of death parameter for each age interval $(x, x + 1]$ is obtained as follows

$$\hat{q}_x^F = 1 - e^{-\hat{\mu}_x} \quad (16)$$

2.2. Graduation Process

A spline is a segmented piecewise polynomial that has flexibility properties. The points of joint integration of these pieces or points that show changes in the behavior of the curve at different intervals are called knots (Fan & Yao, 2005). The natural cubic spline method is the method used to carry out a rough life table graduation that has been obtained in this research. The natural cubic spline (C) function is a cubic polynomial piece on the interval $[a, b]$ which has the form

$$C(x) = \begin{cases} C_0(x) & ; \quad x \in [t_0, t_1] \\ C_1(x) & ; \quad x \in [t_1, t_2] \\ \vdots & \\ C_{n-1}(x) & ; \quad x \in [t_{n-1}, t_n]. \end{cases}$$

For each interval $[t_i, t_{i+1}]$, $C_i(x)$ is defined as

$$\begin{aligned} C_i(x) = {} & \frac{z_{i+1}}{6h_i}(x - t_i)^3 + \frac{z_i}{6h_i}(t_{i+1} - x)^3 \\ & + \left(\frac{y_{i+1}}{h_i} - \frac{h_i}{6}z_{i+1}\right)(x - t_i) \\ & + \left(\frac{y_i}{h_i} - \frac{h_i}{6}z_i\right)(t_{i+1} - x) \end{aligned} \quad (17)$$

where $h_i = t_{i+1} - t_i$. For efficient evaluation, the nested form of $C_i(x)$ is used, namely

$$\begin{aligned} C_i(x) = {} & y_i + B_i(x - t_i) + \frac{z_i}{2}(x - t_i)^2 + \\ & + \frac{(z_{i+1} - z_i)}{6h_i}(x - t_i)^3 \end{aligned} \quad (18)$$

Where $B_i = -\left(\frac{h_i}{6}\right)z_{i+1} - \left(\frac{h_i}{3}\right)z_i + \frac{(y_{i+1} - y_i)}{h_i}$. The coefficients $z_0, z_1, \dots, z_n$ are generated by assuming $b_i = \frac{(y_{i+1} - y_i)}{h_i}$, $u_i = 2(h_{i-1} + h_i)$, $v_i = 6(b_i - b_{i-1})$. Then solve the tridiagonal system equations as follows

$$\begin{cases} z_0 = 0 \\ h_{i-1}z_{i-1} + u_iz_i + h_iz_{i+1} = v_i \,; \ (1 \leq i \leq n - 1) \\ z_n = 0. \end{cases}$$

This can be solved by forward substitution as follows

$$\begin{cases} u_i = \dfrac{h_{i-1}^2}{u_{i-1}} \\ v_i = \dfrac{h_{i-1}v_{i-1}}{u_{i-1}} \end{cases};\ (i = 2,3,\dots,n-1),$$

And backward substitution as follows

$$\begin{cases} z_{n-1} \leftarrow \dfrac{v_{n-1}}{u_{n-1}} \\ z_i \leftarrow \dfrac{v_i - h_i z_{i+1}}{u_i} \end{cases};\ (i = n-2, n-3, \dots, 1).$$

Function (C) is called a natural cubic spline if the domain of (C) is in the interval $[a, b]$ as well as (C), (C'), and (C'') is continuous on the interval $[a, b]$.

3. Result and Discussion

3.1. Rough Life Table Calculation

Based on data calculation from Gegelang Village, rough life table calculation results were obtained using the maximum likelihood method which are presented in Table 1 and Table 2. Table 1 is results of calculating rough life expectancy using the partial data situation method, and Table 2 using the Full Data Situation Method

Table 1. Results of Calculating Rough Life Expectancy Figures using the Partial Data Situation Method

No	x	$_nq_x$	l_x	$_nd_x$	$_nL_x$	T_x	$\dot{e}_x$
1	0	0,00883	100000	883	99558	6221253	62,21
2	5	0,00000	99117	0	495583	6121695	61,76
3	10	0,00795	99117	788	493613	5626112	56,76
4	15	0,00000	98329	0	491644	5132498	52,20
5	20	0,01179	98329	1160	488745	4640855	47,20
6	25	0,00962	97169	934	483510	4152110	42,73
7	30	0,01681	96235	1617	477131	3668600	38,12
8	35	0,00874	94617	827	471020	3191469	33,73
9	40	0,03093	93790	2901	461700	2720450	29,01
10	45	0,07692	90890	6992	436970	2258749	24,85
11	50	0,11254	83898	9442	395886	1821780	21,71
12	55	0,12876	74456	9587	348315	1425894	19,15
13	60	0,14881	64870	9653	300215	1077579	16,61
14	65	0,09804	55216	5413	262548	777364	14,08
15	70	0,20833	49803	10376	223076	514816	10,34
16	75	0,43103	39427	16995	154651	291740	7,40
17	80	0,27778	22433	6231	96586	137089	6,11
18	85	0,00000	16201	16201	40504	40504	2,50

Table 2. The Results of Calculating Rough Life Expectancy Figures using the Full Data Situation Method

No	x	$_nq_x$	l_x	$_nd_x$	$_nL_x$	T_x	$\dot{e}_x$
1	0	0,00346	100000	346	99827	7306570	73,07
2	5	0,00000	99654	0	498272	7206743	72,32
3	10	0,00290	99654	289	497549	6708471	67,32
4	15	0,00000	99365	0	496827	6210921	62,51
5	20	0,00465	99365	462	495672	5714094	57,51
6	25	0,00400	98903	395	493529	5218423	52,76
7	30	0,00607	98508	598	491047	4724894	47,96
8	35	0,00347	97910	340	488703	4233847	43,24
9	40	0,01225	97571	1196	484865	3745144	38,38
10	45	0,02893	96375	2788	474906	3260279	33,83
11	50	0,04744	93587	4440	456836	2785373	29,76
12	55	0,05027	89147	4481	434533	2328537	26,12
13	60	0,06120	84666	5182	410376	1894004	22,37
14	65	0,03819	79484	3036	389832	1483628	18,67
15	70	0,07938	76449	6068	367072	1093795	14,31
16	75	0,15632	70380	11002	324396	726723	10,33
17	80	0,14486	59378	8602	275386	402327	6,78
18	85	0,00000	50776	50776	126941	126941	2,50

Based on the calculation results on table 1 and table 2 different expected figures are obtained. The largest value was obtained using the full data situation method of 73.03 years, while the smallest value was obtained using the partial situation method of 62.21 years. This is because the calculations are carried out using different methods and data conditions. For example, the partial data situation method uses group data, while the full data situation method uses complete data. It is known that the life expectancy for West Lombok Regency in 2015 was 65.1 years. So a method will be chosen that is close to the life expectancy of West Lombok Regency for graduation, namely the partial situation method. The results of rough life table calculations using the partial situation method which have been graduated using the natural cubic spline method are shown in Table 3.

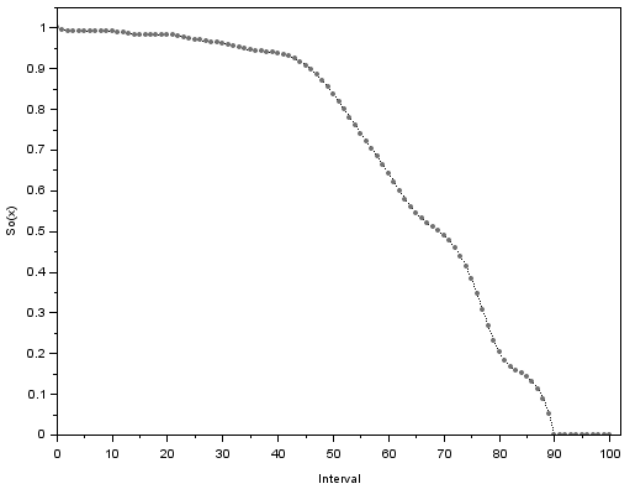
Table 3. The Results of Calculating Rough Life after Graduation using Natural Cubic Spline Method

x	l_x	$_nd_x$	$_nL_x$	T_x	$\dot{e}_x$
0	100000	386	99807	6691751	66,92
1	99614	431	397593	6591944	66,17
5	99183	54	495779	6194351	62,45
10	99129	695	493907	5698573	57,49
15	98434	53	492038	5204666	52,87
20	98381	980	489457	4712627	47,90
25	97402	958	484614	4223170	43,36
30	96444	1580	478270	3738556	38,76
35	94864	894	472084	3260286	34,37
40	93970	2229	464275	2788202	29,67
45	91741	6282	442998	2323927	25,33
50	85459	9391	403817	1880929	22,01
55	76068	9712	356059	1477112	19,42
60	66356	10277	306087	1121053	16,89
65	56079	5978	265450	814967	14,53
70	50101	8674	228820	549517	10,97
75	41427	18209	161612	320698	7,74
80	23218	8099	95841	159086	6,85

x	l_x	${}_nd_x$	${}_nL_x$	T_x	$\dot{e}_x$
85	15119	10029	50521	63245	4,18
90+	5090	5090	12724	12724	2,50

3.2. Graduation Model

The term spline is derived from the thin rods that have been used for a long time by engineers to fit curves through certain points. Spline consist of 3 types, namely linear, quadratic, and cubic. In practice, cubic splines are more often used because they provide a more acceptable approximation. After graduation using the manual cubic spline methode, a graph is obtained as shown in Figure 1.



Figures 1. Graduation graph of partial data situations using the natural cubic spline method

Based on the graph in Figure 1, the x-axis shows the age interval used, namely a five years interval as in the 5 years life table that will be created in this research. Meanwhile, the y-axis shows the survival values as the input value entered into the program. If you pay attention, the graph produced using this method is monotonically decreasing at ages $0 - 60$ and fluctuates at ages after that. Apart from that, after graduation, a life table was obtained as shown in Table 3.

Based on life table, the new life expectancy figure is 66.92 years. The life expectancy obtained after graduation using the natural cubic spline method has a difference of more than four years from before graduation, namely 62.21. Apart from that, the new life expectancy figures obtained are close to the 2015 life expectancy figures for West Lombok.

4. Conclusion

After calculating the rough life table, the life expectancy was 62.21 years using the partial data situation method and the life expectancy was 73.03 years using the full data situation method. Next, the calculation results of the partial data situation method were selected for graduation using the natural cubic spline method. This is because the life expectancy figure produced using the partial data situation method is close to the life ecpectancy figure for Wes Lombok Regency in 2015. After graduation, the new life expectancy figure was 66.92 years.

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