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Mathematical Model of Differential Equations to Population Growth Models with Limited Growth in West Nusa Tenggara Province

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ABSTRACT

Differential equations are often a topic in the field of mathematics which has many applications in mathematical modeling, one of which is population growth. Research on population growth is of course important for an area because the results of this research can be used in issuing policies such as maintaining the availabilityof agricultural land, places to live, and many others. In this study, the mathematical model of differential equations was used to find a population growth model for the West Nusa Tenggara Province, then the model was verified and calculations were carriedout using the Mathematica software. Then a model is generated with the equation $(t) = 3504006\ e^{0{,}012(t-1993)}$ which results in a calculation that the population of NTB will continue to grow so that it is necessary to verify the model which produces a logistics growth model.

Keywords: population growth, West nusa Tenggara Province, Malthusian model, logistic model

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1. Introduction

Mathematics is a science that is universal in nature, where this science is very closely related to everyday life. Mathematics has methods that can be used in every other branch of science which basically aims to develop other sciences (Robbaniyyah, et al, 2024). The methods used are usually quantitative in nature, which will later be studied and developed so that they can be utilized in all aspects of their application. Thus, Mathematics can be said to be applicable because it can be applied in various fields of science with existing methods (Yunianto, 2021).

Differential equations are an interesting part of Mathematics that are often used in solving problems in everyday life (Robbaniyyah, 2022). Differential equations can be applied in various fields, one of which is in the social field. In the social field, differential equations are used to solve problems related to social problems that often occur in people's lives (Rina and Husna, 2019). One example is the problem of population growth. Differential equations can provide exact and analytical solutions which of course will make it easier to solve social problems such as population growth (Pratiwi, 2020).

Population growth is one of the social problem topics that is often studied and researched by experts. The rate of population growth is very influential in various areas of life (Indrianawati and Mahdiyah, 2019). For example, in the government sector, in the economic sector, as well as in the availability of residential land (Nuraeni, 2017). In the government sector, an increase or decrease in the rate of population growth will have a very significant impact on a region. Good and stable population growth will certainly have a positive impact on the development of the area. Apart from that, by looking at the population growth of a region, it can also make it easier for local governments to formulate various public policies for the progress of their own region (Kurniawan, et al., 2017). Then in the economic field, population growth will encourage increased economic growth, but population density has a negative impact on economic growth. In accordance with research conducted by Yunianto in 2021 in Samarinda, the rate of population growth will have an impact on the per capita income of the Samarinda region. However, population density has a negative impact because a 1% increase in population density causes a decrease in economic growth of 22.33%. Furthermore, regarding land availability, research conducted by Indrianawati and Mahdiyah in 2019 stated that the conversion of agricultural

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land caused by population growth had an impact of 12%. This impact does not seem very significant, but if it continues, the availability of agricultural land in the future will decrease and this problem will spread to the availability of food (Pratiwi, 2020).

There are several studies that have been carried out, such as that carried out by Nuraeni (2017) with the title "Application of Differential Equations in Estimating Population Numbers" explaining the differential equations applied by researchers to estimate the population size of an area in a certain period of time. The research was conducted only on the population growth of Maluku Province, which was based on data on the population of Maluku Province from 2010 to 2015.

Research conducted by Pratiwi (2020) entitled "Application of Differential Equations for Logistic Population Models to Estimate Population in Balikpapan City" explains that researchers want to obtain a logistic model for the population growth of Balikpapan City and want to know the estimated population of Balikpapan City in 2025 which will be estimated using a logistic growth model. The data to be used was obtained from the Balikpapan City Central Statistics Agency, which took population data from 2015 to 2018 (Pratiwi,2020).

Research conducted by Kurniawan et al. (2017) with the title "Application of Ordinary Differential Equations Exponential Models to Population Growth in the City of Surabaya" explains the description of the application of differential equations to population growth models in the City of Surabaya using exponential growth models and logistic growth models. The population data that will be used is collected from the Population and Civil Registry Service and BPS. The data used is data from 2011 to 2015 (Kurniawan, 2017).

Research conducted by Ahmad (2019) with the title "Mathematical Modeling Using Differential Equations for Population Growth in Indonesia" explains the case model for population growth in Indonesia. Population growth data was collected from BPS in 1980-2010. This research uses the application of differential equations with exponential population models and logistics population models (Population, 2019).

Based on the research that has been carried out, in this research the application of differential equations will be carried out, namely modeling population growth with limited growth. This model can predict the population size in an area in the coming years. In this case we took the population of West Nusa Tenggara Province as an example.

2. Methodology

In this section, the tool that will be used to help with data processing is Mathematica. The data used in this research is secondary data, namely population census data for West Nusa Tenggara Province from 1993 to 2020 obtained from the West Nusa Tenggara Central Statistics Agency (BPS) website. The research procedure for the application of differential equations to a population growth model with limited growth can be presented in Figure 1 as follows.

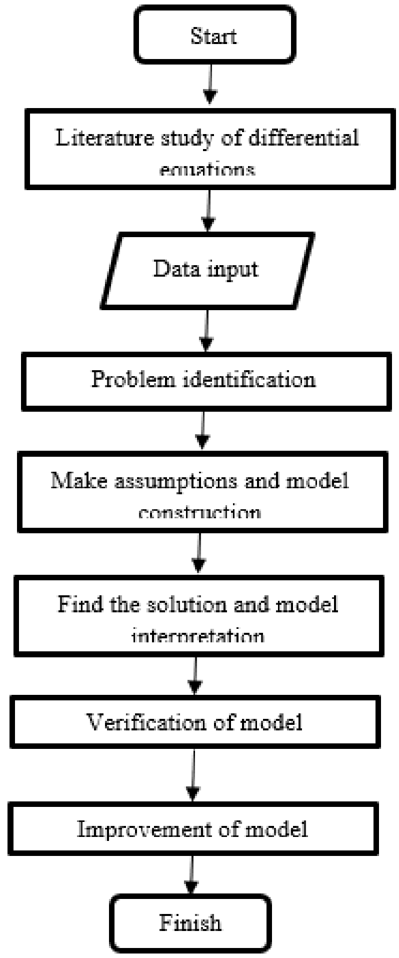

Figure 1. flowchart of research

Based on Figure 1, the section explaining differential equations requires an explanation of the meaning of differential equations, the types of differential equations, namely ordinary differential equations and partial differential equations as well as linear differential equations and nonlinear differential equations, as well as solutions to differential equations, namely using the method separation of variables.

The next step is to input data. The data has been obtained from the West Nusa Tenggara Provincial Statistics Agency website in the form of annual population census data from 1993 to 2020 which will be used to create models and test models. Followed by identifying the problem of population growth in the form of large populations and in the time period t. Then the assumption will be made that population growth is only influenced by birth and death rates, then a population growth model will be constructed using the Malthusian model. After obtaining the model, a model solution will be sought using the variable separation method, then the model will be interpreted and a Malthusian model equation will be formed. After that, verification and improvement of the model were carried out, so that a logistics growth model was obtained as a form of improvement.

3. Result and Discussion

3.1. Identification of Assumptions and Model Construction

Suppose we know the population size at a given time t, for example P_0 at time $t = t_0$. Then, the population number P will be predicted at time $t = t_1$. In other words, we will look for the population function (t) for $t_0 \leq t \leq t_1$ which satisfies $P(t_0) = P_0$. Considering several factors that influence population growth, it is assumed that this model is only influenced by the birth rate and death rate. The human population tends to grow due to increasingly developing science and technology which helps reduce the death rate below the birth rate. Assume that over a short period of time, b is the percentage growth rate and c is the percentage death rate. In other words, the newest population $(t + \Delta t)$ is the old population $P(t)$ plus the number of births minus the number of deaths during the time period Δt.

$$P(t + \Delta t) = P(t) + bP(t)\Delta t - cP(t)\Delta t \qquad (1)$$

From the previous assumption that the average rate of change in a population over a time interval is proportional to the population size, using the temporary rate of change to estimate the average rate of change we get a differential equation model:

$$\frac{dP}{dt} = kP, \;\; P(t_0) = P_0, t_0 \leq t \leq t_1 \qquad (2)$$

with $k > 0$.

From the model in differential equation (2), the solution will be sought using the variable separation method. as follows:

$$\frac{dP}{P} = kdt \qquad (3)$$

by integrating equation (3) is obtained

$$\ln P = kt + C \qquad (4)$$

for some constant C. Apply the condition $P(t_0) = P_0$ in equation (4) to find the value of C, to obtain

$$\begin{aligned} C &= \ln P_0 - kt_0 \\ \ln \frac{P}{P_0} &= k(t - t_0) \end{aligned} \qquad (5)$$

In the final step, both sides are exponentiated to obtain a solution in the form of equation (6) as follows

$$P(t) = P_0 e^{k(t-t_0)} \qquad (6)$$

3.2. Verification of Model

Below we will verify the model by taking a case study example of the population of West Nusa Tenggara (NTB) Province. Previously we have obtained equation (5). The model in equation (6) provides predictions. If the left side of equation (5) is plotted with $(t - t_0)$ that shows in Figure 2 on the following below.

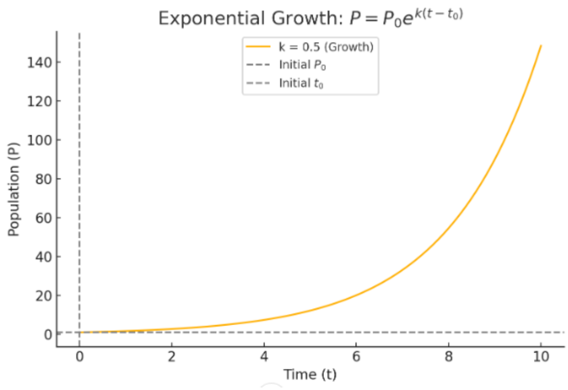

Figure 2. the exponential growth of Equation 5

Then we get a straight line through the origin with slope k. However, the plot results are based on NTB population data from 1993-2020, the model is not coherent. In fact, based on NTB population census data, in 1993 the population was 3,504,006 and in 2000 the population was 3,805,537. In this case, let's say that P_0 is the population in 1993 and 1993 is t_0. Thus, based on equation (6), it can be stated as follows

$$P(2000) = 3504006\, e^{k(2000-1993)}$$
$$k = \left(\ln \frac{3805537}{3504006}\right)/7$$

and we get $k \approx 0.012$. Thus, in percentage over a 7 year period the population of NTB increased at an average rate of 1.2% per year and the shape of the population growth model is as follows.

$$P(t) = 3504006\, e^{0{,}012(t-1993)} \qquad (7)$$

After obtaining the constant value k, the population of NTB in 2010 will be predicted using equation (6) and obtained $P(2010) \approx 4296956$. However, according to the NTB population census in 2010 the population was 4.500.212. Therefore, predictions using this model have an error of 4.52%. In the same way, a prediction of the population for 2020 was made using the same model. Retrieved result $P(2020) \approx 4844804$. However, according to the 2020 census, the population of NTB was 5.125.622 so the error was 5.48%. Then, several predictions were made for future years in Table 1.

Table 1 – Prediction of the population of West Nusa tenggara Province.

Year	Population Prediction
2030	5462500
2050	6944200
2100	12653200
2200	42010000
2300	139478000

Based on the results in Table 1, in the long term the population of West Nusa Tenggara Province will exceed the carrying capacity or exceed the availability of land both for residence and agricultural land. This result certainly does not make sense, so it is necessary to improve the model with limited growth, namely limiting the maximum population to M.

3.3. Improvement of Model

Based on equation (6) there is a growth rate constant k. For example, this constant k is no longer a constant but a function of the population. The more the population grows and the closer it gets to the maximum limit of M, the population growth rate k will decrease. A submodel for k in the form of a linear function can be written as follows

$$k = r(M - P), \; r > 0,$$

where r is a constant. Then substitute equation k into equation (2) to obtain

$$\frac{dP}{dt} = r(M - P)Pdt \tag{8}$$

$$\frac{dP}{P(M-P)} = r\,dt \tag{9}$$

Equation (8) was first introduced by Pierre-Francois Verhulst, known as logistic growth. Furthermore, with a little elaboration, the form corresponding to equation (9) is obtained as follows.

$$\left(\frac{dP}{P} + \frac{dP}{M-P}\right) = rM\,dt \tag{10}$$

The integration process is carried out in equation (10), so that the following results are obtained.

$$\ln P - \ln|M - P| = rMt + C \tag{11}$$

for any constant C. Using the assumption $P(t_0) = P_0$, evaluate the value of C with the case $P < M$ to obtain

$$C = \ln\frac{P_0}{M - P_0} - rM\,t_0 \tag{12}$$

The value of C in equation (12) is substituted into equation (11) and then exponentiated so that it can be written

$$P(t) = \frac{P_0 M e^{rM(t-t_0)}}{M - P_0 + P_0 e^{rM(t-t_0)}} \tag{13}$$

If it is estimated that P at $t \to \infty$, then equation (13) can be written as

$$P(t) = \frac{P_0 M}{M - P_0 + P_0 e^{rM(t-t_0)}} \tag{14}$$

Note that in equation (14), $P(t)$ will approach M when $t \to \infty$. In addition, from equation (8) the second derivative can be obtained which is shown in the equation below.

$$P'' = rP'(M - 2P).$$

P'' will be equal to zero, when $P = M/2$. This shows that the population P reaches half the value of M. The growth of dP/dt is at its fastest growth and starts to decline and goes towards zero. In addition, we can estimate the value of M. The solution to equation (14) can be simulated in the form of Figure 2 for the case $P > M$, where the curve in Figure 2 is called the logistics curve (Giordano, et al., 2014).

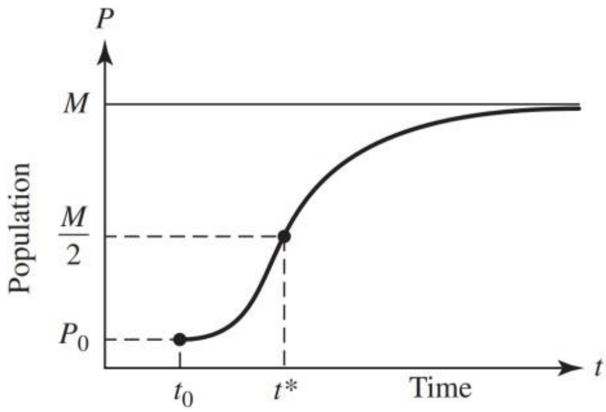

Figure 3. Population Growth Population Logistics Curve

Referring to Table 1, the results correspond to those shown in Figure 3. Then the trend graphics is also relate with Figure 3.

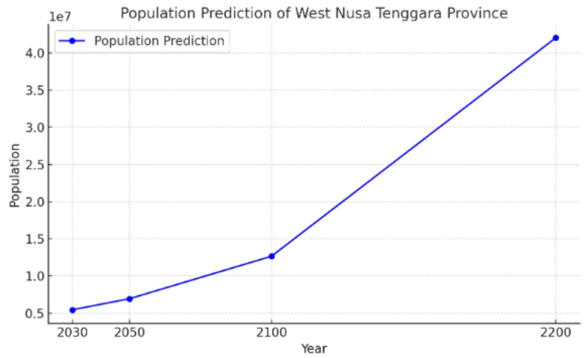

Figure 4. Prediction of the population of West Nusa Tenggara Province.

Based on Figure 4, the prediction of population of West Nusa Tenggara Province get the results that the graph shows a steady increase in population over time, with rapid growth projected after 2100, suggesting exponential-like growth in the population.

4. Conclusion

Based on the results of the research and discussion, it can be concluded that the population growth model for West Nusa Tenggara Province uses mathematical modeling in differential equations to obtain the model in equation (7). Thus it can be seen that the population increase each year has an average rate of 1.2%. Apart from that, a calculation of the population growth of West Nusa Tenggara Province was obtained using Mathematica software which is shown in Table 1. The table shows that the population in West Nusa Tenggara will continue to increase every year so a limited growth model is needed.

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