



Application of the Average Based Fuzzy Time Series Lee Method for Forecasting World Gold Prices

Husnul Khotimah^a, Qurratul Aini^b, Nur Asmita Purnamasari^{c}*

- ^a Program Studi Matematika, Universitas Mataram, Indonesia, Email: husnulk5221@gmail.com
^b Program Studi Matematika, Universitas Mataram, Indonesia, Email: qurratulaini.aini@unram.ac.id
^c Program Studi Statistika, Universitas Mataram, Indonesia, Email: asmitapurnamasari@unram.ac.id

ABSTRACT

Gold is a investment that investors are interested in because it has relatively low risk and gold investment is not affected by inflation. Gold prices always change from time to time, so it is necessary to forecast gold prices as a basis for investors in making decisions. The forecasting method used in the fuzzy time series lee method. The purpose of this research is determine the world prices and determine the accuracy of the gold price forecasting value obtained using fuzzy time series lee method. The results of this research are forecasting gold prices in the period November 20, 2023 of US\$ 63,89/grams and relatively the level of forecasting accuracy based on MAPE value of 0,540091% included in the very good criteria in forecasting gold prices.

Keywords: Fuzzy Time Series Lee, Gold, Mean Absolute Percentage Error

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1. Introduction

As time progresses, many people want to invest to gain profits in the future (Husnan, 2000). Investing is the act of purchasing one or more assets owned over the long term, usually profitably, with the aim of saving money for the future (Sunariyah, 2003).

Gold is one of the precious metals that is in great demand for investment. This is because gold prices tend to be stable and rarely experience price drops. Gold investment can be used as a solution for urgent needs, because gold investment is easier to disburse. Investing in gold has its advantages, for example when the weakening of the US dollar exchange rate triggers an increase in world gold prices which causes the price of gold to become cheaper in other currencies, thereby encouraging an increase in demand for gold. The higher the demand for an item, the more expensive the price. Therefore, gold can be used as a tool to fight inflation and add value in conditions of unstable currency value fluctuations.

To achieve investment goals, before investing, knowledge is needed that can support these activities. One of the important knowledge in investing in gold is price forecasting. Forecasting gold prices is necessary before investing to find out the trend of gold prices in the future. Forecasting is the process of estimating (measuring) the size or amount of something in the future according to historical data (time series) which is analyzed using statistical methods (Sudjana, 1996). Gold price forecasting aims to determine opportunities for gold prices in the future so that it can be used as a consideration by investors to get maximum profits.

Methods that are often used in forecasting include ARIMA, Exponential Smoothing, VARIMA, and others. These methods have the weakness of requiring a lot of historical data and requiring certain assumptions that must be met. One method used to overcome these weaknesses is the fuzzy time series (FTS) method (Wang et al, 2015). FTS is data forecasting that uses fuzzy sets as the basis for forecasting. Forecasting with FTS is forecasting that processes past data patterns and then uses them to predict future data.

* Corresponding author.
e-mail address: asmitapurnamasari@unram.ac.id

FTS has the advantage of not requiring a lot of historical data and not requiring assumptions in making forecasts. Fuzzy Time Series is a data forecasting method that uses fuzzy sets derived from real numbers over a universe of actual data (Elfajar, 2017). Fuzzy time series forecasting reduces the amount of historical data needed by substituting fuzzy sets for the historical data that will be projected.

FTS was first introduced by Song & Chissom. There are several models that can be used on fuzzy time series, including FTS Cheng, FTS Chen, and FTS Lee. The model used is the Lee model because the error rate is the smallest. Fuzzy time series Lee model is a development of FTS Cheng, and FTS Chen to predict a value in the future (Qiu et al, 2011). This model has the same steps as other FTS. However, FTS Lee has a difference with conventional FTS which lies in the formation of a Fuzzy Logical Relationship Group (FLRG). The fuzzy time series approach with an average basis is employed to ascertain the quantity of fuzzy sets and measures the level of error in forecasting using the MAPE value.

1.1. Types of research

This study employs applied research methodology, utilizing the fuzzy time series Lee technique to anticipate global gold prices.

1.2. Data source and Research variable

The secondary data utilized in this study was sourced from the website <https://id.investing.com/currencies/xau-usd-historical-data>. The variable used is the world gold price from January 2 2023 – November 17 2023 with daily gold prices. With a total of 229 observations.

1.3. Work procedures

The research steps carried out in this research start from preparation, data collection, determining the universal set U, determining how many fuzzy sets there are, identifying fuzzy sets and fuzzyfication, determining Fuzzy Logical Relations (FLR) and Fuzzy Logical Relationship Group (FLRG), defuzzification, calculate MAPE value and conclusions.

1.4. Data analysis

The data analysis method used is forecasting using the FTS Lee method, which is one of the developments of the Song and Chissom, Cheng, and Chen models in forecasting a value in the future. The stages carried out in Lee's fuzzy time series forecasting are as follows:

1. Determine the set of speech universes U

$$U = [D_{min} - D_1; D_{max} + D_2] \tag{1}$$

with D_{min} is the minimum data, D_{max} is the maximum data and D_1 and D_2 is an arbitrary positive number determined by the researcher

2. Count the number of fuzzy sets.

In determining the interval length in fuzzy using the average-based method as follows :

- a. Determines the length of the interval U

$$R = (D_{max} + D_2) - (D_{min} - D_1) \tag{2}$$

- b. Calculates the absolute difference and the average absolute difference

$$S_m = |D_{t+1} - D_t| \tag{3}$$

$$mean = \frac{\sum_{t=1}^{n-1} |D_{t+1} - D_t|}{N - 1} \tag{4}$$

where D_t is the data at-t and N is the number of observations.

- c. Calculating interval basis

$$l = \frac{mean}{2} \tag{5}$$

After getting the base interval value, the range value of the base can be used as the interval length of the fuzzy set.

- d. Use the following formula to get the number of fuzzy sets or intervals:

$$n = \frac{R}{l} \tag{5}$$

n is the number of intervals, R is the duration of the U-interval, and l is the interval base.

Then U is divided into n intervals with the same interval length (l), with the interval length obtained by the formula:

$$\begin{aligned} u_1 &= (D_{min} - D_1; D_{min} - D_1 + l) \\ u_2 &= (D_{min} - D_1 + l; D_{min} - D_1 + 2l) \\ u_3 &= (D_{min} - D_1 + 2l; D_{min} - D_1 + 3l) \\ &\vdots \\ u_n &= (D_{min} - D_1 + (k - 1)l; D_{min} - D_1 + nl) \end{aligned} \tag{6}$$

- e. Determining the middle value of a fuzzy set (m_i)

$$m_i = \frac{lower\ limit\ u_i + upper\ limit\ u_i}{2} \tag{7}$$

3. Define the degree of membership of the fuzzy set to A_i and perform fuzzification on the actual data.

In general, a fuzzy set can be thought of as a number base with ambiguous bounds. A fuzzy set A_i of U with generic membership degrees is described as follows if the universe of

discourse (U) is a set of universes $U = \{u_1, u_2, \dots, u_n\}$, where u_j is a possible value of U.

$$A_i = \frac{\mu_{A_i}(u_1)}{u_1} + \frac{\mu_{A_i}(u_2)}{u_2} + \frac{\mu_{A_i}(u_3)}{u_3} + \dots + \frac{\mu_{A_i}(u_n)}{u_n} \quad (8)$$

with $\mu_{A_i}(u_1)$ is a degree of membership that has a range $[0,1]$ and $1 \leq i \leq n$.

Fuzzification is the process of leveraging membership values kept in a fuzzy knowledge base to convert system inputs with firm (numerical) values into linguistic variables.

4. Create a Fuzzy Logical Relationship (FLR) based on actual data.

FLR connects the relations between linguistic value determined based on the fuzzification table obtained previously.

5. Creating a Fuzzy Logical Relation Group (FLRG) Lee model.

FLRG is carried out by grouping fuzzyfications that have the same current state and then grouping them together in the next state. In FTS Lee, all FLRs are grouped into interconnected FLRG. Example $A_1: A_1 \rightarrow A_2, A_1 \rightarrow A_2, A_1 \rightarrow A_3$. The three FLRs can be grouped into: $A_1 \rightarrow A_2, A_2, A_3$.

6. Defuzzification

Defuzzification is changing linguistic variables obtained from fuzzy logic rules into firm value using membership value that correspond to the time the fuzzy fication was carried out. The rules for performing defuzzification on the Lee model are:

Rule 1

The following forecasting results are obtained if the fuzzy fication result in year t is A_j and there is a fuzzy fication that does not have a fuzzy logic relationship, such as $A_i \rightarrow \emptyset$, where the middle value of u_j is m_k and the maximum value of A_i 's membership value is in the interval u_j .

$$F_t = m_k \quad (9)$$

Rule 2

The following forecasting results are produced if the fuzzyfication result in year t is A_j , in FLRG there is only one FLR (eg $A_i \rightarrow A_j$ where A_i and A_j are fuzzyfications, the maximum value of A_j membership value is in the interval u_j , and the middle value of u_j is m_l).

$$F_t = m_l \quad (10)$$

Rule 3

If A_m, A_n, \dots, A_o are the results of fuzzyfication in year t, then FLR(p) in FLRG has several value such as $A_i, A_j \rightarrow A_m, A_m, A_n, A_n, \dots, A_o$ where $A_m, A_m, A_n, A_n, \dots, A_o$ represents fuzzyfication, and the maximum membership value is $A_m, A_m, A_n, A_n, \dots, A_o$, when $m_l, m_l, m_m, m_m, \dots, m_n$, is the

middle value in the interval $u_m, u_m, u_n, u_n, \dots, u_o$, the forecasting results are as follows:

$$F_t = \frac{2}{p} m_m + \frac{2}{p} m_n + \dots + \frac{1}{p} m_o \quad (11)$$

7. Calculating Forecasting Accuracy Values

The average of MAPE is the overall percentage error (difference) between actual data and forecast data. The following is the MAPE formula:

$$MAPE = \frac{1}{N} \sum_{t=1}^n \frac{|D_t - F_t|}{D_t} \times 100 \% \quad (12)$$

2. Result and Discussion

2.1. Determining the Set of the Universe U Talk

World Gold Price Data for the period 2 January 2023 to 17 November 2023 has the lowest gold price of US\$ 58,23/gram and the highest gold price of US\$ 65.95/gram. Based on equation 1, the value of D_1 and D_2 are any positive numbers. Researchers determined $D_1=0,2$ and $D_2=0,7$. Based on equation 1, the set of conversation universes (U) is as follows:

$$\begin{aligned} U &= [D_{min} - D_1; D_{max} + D_2] \\ &= [58,23 - 0,2; 65,95 + 0,7] \\ U &= [58,03; 66,65] \end{aligned}$$

2.2. Determining the Number of Fuzzy Sets

In determining the interval length in fuzzy using the average-based method as follows :

a. Determines the length of the interval U

$$\begin{aligned} R &= (D_{max} + D_2) - (D_{min} - D_1) \\ &= (65,95 + 0,7) - (58,23 - 0,2) \\ &= (66,65 - 58,03) \\ R &= 8,62 \end{aligned}$$

b. Calculates the absolute difference and the average absolute difference

Table 1. Absolute Difference in Actual Data

No	US\$/Gram	$ D_{t+1} - D_t $
1	58,64	0,51
2	59,15	0,47
3	59,62	0,67
4	58,95	1,05
5	59,99	0,19
6	60,18	0,18
7	60,36	0,03
⋮	⋮	⋮
227	62,99	0,70
228	63,69	0,03
229	63,67	-
Σ		85,64

Based on Table 1, the absolute difference in actual data is 85,64. The sum of the absolute differences in the data is used to calculate the absolute average value of each data. Calculation of the average absolute difference for each data

uses equation 4. The following is the calculation of the average value of the absolute difference:

$$\begin{aligned} \text{mean} &= \frac{\sum_{t=1}^{n-1} |D_{t+1} - D_t|}{n-1} \\ &= \frac{85,64}{229-1} \\ &= 0,38 \end{aligned}$$

c. Calculating interval basis

The results of the average absolute difference for each data are used to calculate the fuzzy set interval basis using equation 5. The following is the calculation of the fuzzy set interval basis:

$$\begin{aligned} l &= \frac{\text{mean}}{2} \\ &= \frac{0,38}{2} \\ &= 0,19 \approx 0,2 \end{aligned}$$

Based on the results above, the basic interval value obtained is 0,19. The interval base value is 0,19, included in the interval base of 0,1 with rounding of the interval length to 0,2.

d. Count the number of fuzzy sets

The interval basis is used to calculate the number of fuzzy sets using equation 6. The following is the calculation of the number of fuzzy sets:

$$\begin{aligned} n &= \frac{R}{l} \\ &= \frac{8,62}{0,2} \\ &= 43,1 \approx 43 \end{aligned}$$

Based on the calculation of the number of fuzzy sets, the results obtained were 43 fuzzy sets. The fuzzy set has the same interval length, namely 0,2, then $U=(58,03;66,65]$ is partitioned into 43 sets of the same length, namely u_j where $j=1,2,\dots,43$. Based on this partition, then The fuzzy set and the middle value formed are as follows:

Table 2. Middle Value of Fuzzy Sets

Intervals	Lower limit	Upper limit	Middle value
u_1	58,03	58,23	58,13
u_2	58,23	58,43	58,33
u_3	58,43	58,63	58,53
u_4	58,63	58,83	58,73
u_5	58,83	59,03	58,93
\vdots	\vdots	\vdots	\vdots
u_{39}	66,63	65,83	65,73
u_{40}	66,83	66,03	65,93
u_{41}	66,03	66,23	66,13
u_{42}	66,23	66,43	66,33
u_{43}	66,43	66,63	66,53

Based on Table 2, the middle value of the 1st to 43 rd fuzzy set is obtained using equation 7. The following is an example of calculating the middle value of the 1st fuzzy set (m_1):

$$m_i = \frac{\text{lower limit } u_i + \text{upper limit } u_i}{2}$$

$$\begin{aligned} &= \frac{58,03 + 58,23}{2} \\ &= 58,13 \end{aligned}$$

2.3. Determining the Degree of Fuzzy Set Membership of A_i in the Fuzzyfication Process

Determining the degree of fuzzy set membership to A_i is based on the 43 fuzzy sets formed in the previous stage. It is assumed that the fuzzyfication value of the linguistic variables of the Gold Price are $A_1, A_2, A_3, \dots, A_{43}$. Each fuzzy set u_j where $j = 1, 2, \dots, 43$ is defined against A_i using equation 8. The following defines the degree of membership of a fuzzy set to A_i :

$$\begin{aligned} A_1 &= 1/u_1 + 0,5/u_2 + 0/u_3 + \dots + 0/u_{43} \\ A_2 &= 0,5/u_1 + 1/u_2 + 0,5/u_3 + \dots + 0/u_{43} \\ A_3 &= 0/u_1 + 0,5/u_2 + 1/u_3 + \dots + 0/u_{43} \\ &\vdots \\ A_{42} &= 0/u_1 + 0/u_2 + 0/u_3 + \dots + 0,5/u_{41} + 1/u_{42} + 0,5/u_{43} \\ A_{43} &= 0/u_1 + 0/u_2 + 0/u_3 + \dots + 0,5/u_{42} + 1/u_{43} \end{aligned}$$

Based on defining the degree of fuzzy set membership to A_i , fuzzyfication results are obtained.

Table 3. Fuzzyfication of Actual Data

No	US\$/Grams	Fuzzyfication
1	58,64	A4
2	59,15	A6
3	59,62	A8
\vdots	\vdots	\vdots
227	62,99	A25
228	63,69	A29
229	63,67	A29

Determining the fuzzyfication value is carried out by defining the data into appropriate intervals, for example in the first data in this study, namely in the period 2 January 2023 with a value of 58,64, the data falls into the base interval u_4 (58,63; 58,83]. In the fuzzyfication process, the data will be converted into linguistic value A4 and so on until data 229.

2.4. Determining Fuzzy Logical Relationship (FLR) from Actual Data

FLR is an activity carried out by connecting relations between linguistic variables determined based on the fuzzyfication table obtained in Table 3. The FLR results can be seen in Table 4.

Table 4. Fuzzy Logical Relationship (FLR)

No	US\$/Gram	Current State	FLR	Next State
1	58,64	-	→	A_4
2	59,15	A_4	→	A_6
3	59,62	A_6	→	A_8
4	58,95	A_8	→	A_5
5	59,99	A_5	→	A_{10}
6	60,18	A_{10}	→	A_{11}
\vdots	\vdots	\vdots	\vdots	\vdots
227	62,99	A_{26}	→	A_{25}

No	US\$/Gram	Current State	FLR	Next State
228	63,69	A_{25}	\rightarrow	A_{29}
229	63,67	A_{29}	\rightarrow	A_{29}
		A_{29}		

Based on Table 4, determining FLR from actual data is symbolized by $D_{t-1} \rightarrow D_t$. For example, data number 2 is the current state (D_{t-1}) with a fuzzyfication value of A_4 . Data number 3 is the next state (D_t) with a fuzzyfication value of A_6 . The FLR result formed is $A_4 \rightarrow A_6$. The next FLR has the same steps as FLR in number 2 with number 3.

2.5. Determining the Fuzzy Logic Relationship Group (FLRG)

FLRG is carried out by grouping fuzzyfications that have the same 1 current state, namely D_{t-1} and then grouping them into one group in the next state. FLRG results can be seen in Table 5.

Table 5. Fuzzy Logical Relationship Group (FLRG)

Group	FLRG
1	$A_1 \rightarrow A_2$
2	$A_2 \rightarrow A_2, A_4, A_5$
3	$A_3 \rightarrow A_1, A_3, A_3, A_5$
4	$A_4 \rightarrow A_3, A_3, A_6, A_6$
⋮	⋮
36	$A_{38} \rightarrow A_{32}$
37	$A_{40} \rightarrow A_{35}, A_{40}$

Based on Table 5, all FLRs formed in Table 5 are grouped into interconnected FLRGs. For example, the FLRG formed in group 3 is $A_3 \rightarrow A_1, A_3 \rightarrow A_3, A_3 \rightarrow A_3$, and $A_3 \rightarrow A_5$. The 4 FLRs are grouped into 1 FLRG, namely $A_3 \rightarrow A_1, A_3, A_3, A_5$. FLRG in the next group has the same steps.

2.6. Defuzzification

The forecasting value is calculated based on the FLRG obtained in the previous step, then defuzzification is carried out or the process of changing linguistic variables into firm (numerical) value using FTS Lee. For example, A_2 forms FLRG $A_2 \rightarrow A_2, A_4, A_5$ with A_2 using the middle value $u_2(m_2)$, A_4 using the middle value $u_4(m_4)$, and A_5 using the middle value $u_5(m_5)$. By using equation 10, namely $\frac{(m_2+m_4+m_5)}{3}$. Based on FLRG, the defuzzification value obtained from the Gold Price data can be seen in Table 6.

Table 6. Defuzzification

FLRG	Nilai Peramalan
$A_1 \rightarrow A_2$	58,33
$A_2 \rightarrow A_2, A_4, A_5$	58,66
$A_3 \rightarrow A_1, A_3, A_3, A_5$	58,53
$A_4 \rightarrow A_3, A_3, A_6, A_6$	58,83
$A_5 \rightarrow A_4, A_{10}, A_{10}, A_{11}$	59,68
$A_6 \rightarrow A_5, A_6, A_6, A_6, A_6, A_8, A_9$	59,24
⋮	⋮
$A_{29} \rightarrow A_{25}, A_{27}, A_{27}, A_{29}, A_{29}, A_{29}, A_{30}, A_{30}, A_{33}, A_{34}, A_{35}$	63,89
$A_{30} \rightarrow A_{28}, A_{29}, A_{29}, A_{29}, A_{30}, A_{30}, A_{31}, A_{32}, A_{32}$	63,95

FLRG	Nilai Peramalan
$A_{31} \rightarrow A_{28}, A_{28}, A_{29}, A_{33}, A_{33}$	63,97
$A_{32} \rightarrow A_{30}, A_{31}, A_{34}$	64,26
$A_{33} \rightarrow A_{29}, A_{30}, A_{31}, A_{31}$	63,98
$A_{34} \rightarrow A_{34}, A_{35}, A_{38}, A_{40}$	65,28
$A_{35} \rightarrow A_{30}, A_{33}, A_{35}, A_{35}, A_{35}, A_{37}$	64,73
$A_{37} \rightarrow A_{34}, A_{37}$	65,03
$A_{38} \rightarrow A_{32}$	64,33
$A_{40} \rightarrow A_{35}, A_{40}$	65,43

Based on Table 6, the forecasting value of FLRG group 1 is 58,33. This value is obtained because the FLR formed in FLRG is only 1, namely $A_1 \rightarrow A_2$. So, defuzzification of the forecasting value in group 1 uses equation 11. Based on this equation, the forecasting value used is the middle value of u_2 , namely 58,33. Next, the calculation for A_2 is the middle value of $A_2+A_4+A_5$ divided by 3 (the number of constants in that variable). So the forecasting result A_2 is 58,66. And the same steps on the others.

2.7. Calculating the Level of Forecasting Accuracy

Forecasting accuracy functions to see the level of accuracy of forecasting results. To determine forecasting accuracy, the MAPE value is used. In this research, the MAPE value can be calculated using equation 12.

$$\begin{aligned}
 MAPE &= \frac{1}{N} \sum_{t=1}^n \frac{|D_t - F_t|}{D_t} \times 100 \\
 &= \frac{1}{229} (1,231409) \times 100\% \\
 &= 0,540091\%
 \end{aligned}$$

The MAPE value was obtained at 0,540091%. Because the MAPE number is less than 10%, it indicates that the outcomes of use FTS Lee to estimate world gold prices are rather excellent.

2.8. Forecasting Results

Forecasting using FTS Lee to see the estimated Gold Price for the period 20 November 2023, the forecast value obtained by looking at the FLR formed, the FLR formed is then matched with the FLRG formed in this forecasting case.

Tabel 8. Hasil Peramalan

Period	Data	FLRG	Hasil Peramalan
Nov-17-23	63,67	$A_{29} \rightarrow A_{29}$	63,89
Nov-20-23		$A_{29} \rightarrow A_{25}, A_{27}, A_{27}, A_{29}, A_{29}, A_{29}, A_{30}, A_{30}, A_{33}, A_{34}, A_{35}$	63,89

3. Conclusion and Advice

3.1. Conclusion

Based on the discussion above, It is possible to conclude the following:

1. Based on the results of gold price forecasting using Fuzzy Time Series Lee in period January 2 to November

17 2023, so results were obtained for the next period that on November 20 2023, is US\$ 63,89/gram or in rupiah is IDR 986,781.05 at the price The dollar exchange rate on November 20, 2023 is IDR 15,445. The real gold price on November 20 2023 is US\$ 63,58/gram or in rupiah is IDR 981.993.1.

2. Based on the results of the analysis, it was found that the accuracy of the forecasting value or Mean Absolute Percentage Error (MAPE) was 0.540091%, so it can be concluded that the Fuzzy Time Series Lee model has very good criteria because it has a MAPE value of less than 10%.

3.2. Advice

For future researchers, it is hoped that it can be used as a reference and it is recommended that Lee's FTS method be developed again by using other formulas apart from the Average Based formula. The hope is that it can produce better forecasting accuracy.

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