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Hyper-Wiener and Szeged Indices of non-Coprime Graphs of Modulo Integer Groups

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A B S T R A C T

The non-coprime graph of the integer modulo group is a graph whose vertices represent the elements of the integer modulo group, excluding the identity element. Two distinct vertices are adjacent if and only if their orders are not relatively prime. This study explores two topological indices, the Hyper-Wiener index and the Szeged index, in the non-coprime graph of the integer modulo-n group. The results reveal that these indices are equal when the order is a prime power but differ when the order is the product of two distinct prime numbers. This research provides new insights into the patterns and characteristics of these indices, contributing to a broader understanding of the application of graph theory to abstract group structures.

Keywords: non-coprime graph, Hyper-Wiener index, Szeged index

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1. Introduction

Graph theory is a well-established branch of mathematics with applications in various fields. It was introduced by Leonhard Euler in 1736 and has numerous applications in solving real-life problems, such as route or shortest path problems, scheduling problems, and chemical structure representation problems [1]. In the context of chemical structure representation, graph theory is connected to another branch of mathematics called topological indices. Graph theory is used to represent chemical compounds and molecules in chemistry. Topological indices are numerical values related to chemical bonding that express relationships between chemical structures [2].

There are various types of topological indices studied in mathematics, such as Wiener index, Zagreb index, Gutman index, Szeged index and hyper-Wiener index. Wiener index is the first topological index that developed the notion[3]. As the Wiener index was introduced in 1947, the hyper-Wiener and Szeged indices were introduced as generalize the Wiener index. These topological indices are not only used for chemical structures but can also characterize graphs representing abstract concepts [4].

Many studies have been conducted on topological indices in abstract graph concepts, such as the research by Putra titled "The power graph representation for integer modulo group with power prime order". That research considered the general formula of the power graph for the integer modulo ring of order a prime power p^k, where p is a prime number and k is a positive integer, was considered[5]. He gives the general formula to calculate some topological indices such as the Zagreb index, the Harmonic index, the Gutman index, and the Wiener index [6]. Similar results are given by Asmarani, who formulates how to calculate the Zagreb index, the Wiener index, and the Gutman index of the power graph for the dihedral group [7]. Husni provides formulas for calculating the Gutman index and Harmonic index for the coprime graph of the integer modulo group [8].

Apart from coprime graphs of the integer modulo group, there is exist the non-coprime graph of the integer group. Non-coprime graphs of modulo integer groups are graph representations of the modulo integer groups, where the nodes represent elements of the modulo integer group except for the identity element. Two nodes in a non-coprime graph of modulo integer groups are adjacent if the orders of the two nodes are not coprime. These non-coprime graphs exhibit unique characteristics in their structures. Nurhabibah provides characterizations and numerical invariants of the non-coprime graph for generalized quaternion groups [9]. Masriani, et al. (2020) offer characterizations of the graph if the order of dihedral groups is a prime power or if the order is the multiplication of two primes. In a similar vein, Misuki, et al. (2020) provides the same characterizations but for a different group called the Dihedral group. Therefore, it is intriguing to investigate the topological indices of non-Coprime Graphs of Integers Modulo n Group.

Based on the properties of non-coprime graphs of modulo integer groups mentioned in previous research, this study aims to investigate the characteristics of the hyper-Wiener and Szeged indices within these graphs. The hyper-Wiener is a topological index that calculated based on distance between all vertex on graph. Similarly, the Szeged index also calculated based on distance vertex but if and only if two vertex are adjacent. The hyper-Wiener and Szeged indices were chosen due to their development from the Wiener index, which has been extensively studied by numerous researchers.

By analyzing the topological indices of non-coprime graphs of modulo integer groups, this research expected a patterns or characteristics can be discovered, contributing to a better understanding of the research being discussed. Thus, this study is expected to make a significant contribution to the development of graph theory and its applications in the context of non-coprime graphs of modulo integer groups.

2. Result and Discussion

Some of the results that have been discovered by previous researchers and serve as the fundamental basis of this study include, among others, various key findings that provide essential theoretical foundations, supporting concepts, and crucial references that shape the direction and development of this research.

Definition 2.1 [10] For every positive integer n, the set of positive integers $\mathbb{Z}^+$ can be divided into n cells based on the remainder if a positive integer is divided by n, ranging from $0, 1, 2, ..., n-1$. The set of residue cells modulo n under addition forms a group known as the modulo n integer group. The modulo n integer group is expressed as $\mathbb{Z}_n = \{0, 1, 2, ..., n-1\}$.

We can define the graph representation of $\mathbb{Z}_n$ to visualize its abstract concept. Definition 2.2 given the definition of non-coprime graph representation of $\mathbb{Z}_n$

Definition 2.2 [11] Non-coprime graphs represent the modulo integer groups, with nodes corresponding to group elements except for the identity element. Two distinct vertices x, y are adjacent if and only if $(|x|, |y|) \neq 1$. The non-coprime graph of the modulo n integer group is denoted by $\overline{\Gamma}_{\mathbb{Z}_n}$

Masriani, et al., (2020) also gave a property of the non-coprime graph of modulo integer group as stated in Theorem 2.1.

Theorem 2.1. *[12] The non-coprime graph of the modulo n integer group for $n = p^m$, where p is a prime number and m is a positive integer, is a complete graph $K_{(n-1)}$.*

Klein, et al. (1995) gave a definition of the hyper-Wiener index of graph representation cycle-containing structure molecules. Definition 2.3 stated the definition of the hyper-Wiener index of non-coprime graph of the modulo n integer group.

Definition 2.3 [13] The hyper-Wiener index of the non-coprime graph of the modulo n integer group is denoted as $WW(\overline{\Gamma}_{\mathbb{Z}_n})$ and is defined as follows:

$$WW(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2} \sum_{u,v \in V(\overline{\Gamma}_{\mathbb{Z}_n})} (d(u,v) + d^2(u,v))$$

where $d(u, v)$ represents the distance from vertex u to vertex v.

Gutman and Dobrynin (1998) gave a definition of The Szeged index of an arbitrary graph. Here, the definition of The Szeged index of the non-coprime graph of the modulo n integer group stated in the Definition 2.4

Definition 2.4 [14] The hyper-Wiener index of the non-coprime graph of the modulo n integer group is denoted as $Sz(\overline{\Gamma}_{\mathbb{Z}_n})$ and is defined as follows:

$$Sz(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2} \sum_{uv \in E(\overline{\Gamma}_{\mathbb{Z}_n})} (n_u(uv|\overline{\Gamma}_{\mathbb{Z}_n})(n_v(uv|\overline{\Gamma}_{\mathbb{Z}_n})$$

for $n_u(uv|\overline{\Gamma}_{\mathbb{Z}_n})$ represents the number of vertices in $\overline{\Gamma}_{\mathbb{Z}_n}$ that are closer to vertex u than to vertex v, and $n_v(uv|\overline{\Gamma}_{\mathbb{Z}_n})$ represents the number of vertices in $\overline{\Gamma}_{\mathbb{Z}_n}$ that are closer to vertex v than to vertex u, for any $u, v \in V(\overline{\Gamma}_{\mathbb{Z}_n})$, where $\overline{\Gamma}_{\mathbb{Z}_n}$ is the set of all vertices

Based on the literature review conducted, several theorems related to the hyper-Wiener and Szeged indices of the non-coprime graph of the integer modulo n group have been obtained.

Theorem 2.2. *Given $\mathbb{Z}_n$ with n being a prime power, the hyper-Wiener index of the non-coprime graph of $\mathbb{Z}_n$ is $WW(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}(n-1)(n-2)$*

Proof. Based on Theorem 2.1, since $\overline{\Gamma}_{\mathbb{Z}_n}$ is a complete graph $K_{(n-1)}$, for any $x, y \in V(\overline{\Gamma}_{\mathbb{Z}_n})$, $d(x, y) = 0$ if $x = y$ and $d(x, y) = 1$ if $x \neq y$. Thus, $\frac{1}{2}[d(x,y) + d^2(x,y)] = 1$ if $x \neq y$. Furthermore, since $\overline{\Gamma}_{\mathbb{Z}_n}$ has $n-1$ distinct vertices, the Hyper-Wiener index of $\overline{\Gamma}_{\mathbb{Z}_n}$ is $WW(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}(n-1)(n-2)$. □

Theorem 2.3. *Given $\mathbb{Z}_n$ with n being a prime power, the Szeged index of the non-coprime graph of $\mathbb{Z}_n$ is $Sz(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}(n-1)(n-2)$*

Proof. Based on Theorem 2.1, since $\overline{\Gamma}_{\mathbb{Z}_n}$ is a complete graph $K_{(n-1)}$, so for any $x, y \in V(\overline{\Gamma}_{\mathbb{Z}_n})$, the number of vertices that are closer to x than to y is 1, vertex x itself, and vice versa. Thus, $n_x(xy|\overline{\Gamma}_{\mathbb{Z}_n}) n_y(xy|\overline{\Gamma}_{\mathbb{Z}_n}) = 1$. Furthermore, since $\overline{\Gamma}_{\mathbb{Z}_n}$ has $n-1$ distinct vertices, then $Sz(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}(n-1)(n-2)$ □

Theorem 2.4. *For every $x, y \in V(\overline{\Gamma}_{\mathbb{Z}_n})$, where $x \notin <p_1>$ and $y \notin <p_2>$ or vice versa, if $n = p_1p_2$ where p_1 and p_2 are two distinct prime numbers, then $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$, where $E(\overline{\Gamma}_{\mathbb{Z}_n})$ is the set of all edges.*

Proof. Let $V(\overline{\Gamma}_{\mathbb{Z}_n})$ be divided into subsets $A = \{p_1, 2p_1, ..., n - p_1\}$, $B = \{p_2, 2p_2, ..., n - p_2\}$, and $C = \mathbb{Z}_n \backslash (A, B, \{0\})$. Next will be showed adjacency for every $x, y \in V(\overline{\Gamma}_{\mathbb{Z}_n})$ in six cases.

Case 1. If $x, y \in A$, because $n = p_1p_2$, then $|x| = |y| = p_2$, thus $(|x|, |y|) = p_2$. Since p_2 is prime, $(|x|, |y|) \neq 1$, thus $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$.

Case 2. If $x, y \in B$, then $|x| = |y| = p_1$, so $(|x|, |y|) = p_1 \neq 1$, thus $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$.

Case 3. If $x, y \in C$. Suppose that l is the order of x. Since l does not divide p_1 and p_2, it follows that $|x| = |y| = n$, so $(|x|, |y|) = n \neq 1$, thus $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$.

Case 4. If $x \in A$ and $y \in C$. Analogous by the proof of Case 1 and Case 3, we have $(|x|, |y|) = p_2$, so $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$.

Case 5. If $x \in B$, $y \in C$. Following the explanation of Case 2 and Case 3, $(|x|, |y|) = p_1$, so $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$.

Case 6.If $x \in A$ and $y \in C$, then $(|x|, |y|) = (p_2, p_1) = 1$, because p_1 and p_2 are two distinct prime numbers. So $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$. Thus, it is proven.

□

Theorem 2.5. *Let $\mathbb{Z}_n$ be the modulo integer group with $n = p_1p_2$, where p_1 and p_2 are two distinct prime numbers. Then* $WW(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}[(p_2 - 1)(p_2 - 2) + (p_1 - 1)(p_1 - 2) + (n - p_2 - p_1)(n - p_2 - p_1 + 1)] + (n - p_2 - p_1 + 1)(p_1 + p_2 - 2) + 3(p_1 - 1)(p_2 - 1)$

Proof. Let $\mathbb{Z}_n$ be the modulo integer group with $n = p_1p_2$, for p_1 and p_2 are two distinct prime numbers. Then $V(\overline{\Gamma}_{\mathbb{Z}_n})$ is the union of A$A = \{p_1, 2p_1, ..., n - p_1\}$, $B = \{p_2, 2p_2, ..., n - p_2\}$, and $C = \mathbb{Z}_n \backslash (A, B, \{0\})$. We consider six cases.

Case 1. If $x, y \in A$. Based on Theorem 4.3, $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$. Thus, $\frac{1}{2}[d(x, y) + d^2(x, y)] = 1$. Since $A = \{p_1, 2p_1, ..., n - p_1\}$, then A has $p_2 - 1$ elements. Hence, the hyper-Wiener index of this case is $\frac{1}{2}(p_2 - 1)(p_2 - 2)$.

Case 2. If $x, y \in B$. Based on Theorem 4.3, $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$. Thus $\frac{1}{2}[d(x, y) + d^2(x, y)] = 1$. Since $B = \{p_2, 2p_2, ..., n - p_2\}$, then B has $p_1 - 1$ elements. Hence, the hyper-Wiener index of this case is $\frac{1}{2}(p_1 - 1)(p_1 - 2)$.

Case 3. If $x, y \in C$. Based on Theorem 4.3, $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$. Thus, $\frac{1}{2}[d(x, y) + d^2(x, y)] = 1$. Since $C = \mathbb{Z}_n \backslash (A, B, \{0\})$, then C has $n - (p_2 - 1) - (p_1 - 1) - 1 = n - p_2 - p_1 + 1$ elements. Hence, the hyper-Wiener index of this case is $\frac{1}{2}(n - p_2 - p_1)(n - p_2 - p_1 + 1)$.

Case 4. If $x \in A$ and $y \in C$. Based on Theorem 4.3, $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$. Since A has $p_2 - 1$ elements and C has $n - p_2 - p_1 + 1$ elements, the hyper-Wiener index in this case is $(p_2 - 1)(n - p_2 - p_1 + 1)$.

Case 5. If $x \in B$, $y \in C$. Based on Theorem 4.3, $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$. Thus, $\frac{1}{2}[d(x, y) + d^2(x, y)] = 1$. Since B has $p_1 - 1$ elements and C has $n - p_2 - p_1 + 1$ elements, the hyper-Wiener index in this case is $(p_1 - 1)(n - p_2 - p_1 + 1)$.

Case 6. If $x \in A$ and $y \in B$. Based on Theorem 4.3, $xy \notin E(\overline{\Gamma}_{\mathbb{Z}_n})$, but there exist $w \in C$ such that $xw, yw \in E(\overline{\Gamma}_{\mathbb{Z}_n})$. Thus, the distance between x and y is 2. Therefore, $\frac{1}{2}[d(x, y) + d^2(x, y)] = 3$. Since A has $p_2 - 1$ elements and B has $p_2 - 1$ elements, the hyper-Wiener index in this case is $3(p_1 - 1)(p_2 - 1)$.

By summing up the hyper-Wiener indices from each case, the following result is obtained: $WW(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}[(p_2-1)(p_2-2)+(p_1-1)(p_1-2)+(n-p_2-p_1)(n-p_2-p_1+1)]+(n-p_2-p_1+1)(p_1+p_2-2)+3(p_1-1)(p_2-1)$.

□

Theorem 2.6. *Let $\mathbb{Z}_n$ be the modulo integer group with $n = p_1p_2$, where p_1 and p_2 are two distinct prime numbers. Then $Sz(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}((p_1-2)(p_1-1)+(p_2-2)(p_2-1)+(n-p_2-p_1)(n-p_2-p_1+1))+p_1(p_2-1)(n-p_2-p_1+1)+p_2(p_1-1)(n-p_2-p_1+1)$.*

***Proof*.** Let $\mathbb{Z}_n$ be the modulo integer group with $n = p_1p_2$, where p_1 and p_2 are two distinct prime numbers. Then $V(\overline{\Gamma}_{\mathbb{Z}_n})$ is a union of $A = \{p_1, 2p_1, ..., n-p_1\}$, $B = \{p_2, 2p_2, ..., n-p_2\}$, and $C = \mathbb{Z}_n \backslash (A, B, \{0\})$.

Based on adjacency properties on Theorem 4.3, let's consider five cases.

Case 1. For $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$, if $x, y \in A$. The number of vertices closer to x than to y is 1, i.e., the vertex x itself and vice versa. Thus, $n_x(xy|\overline{\Gamma}_{\mathbb{Z}_n})n_y(xy|\overline{\Gamma}_{\mathbb{Z}_n}) = 1$. Since A has p_2-1 elements, the Szeged index of this case is $\frac{1}{2}(p_2-1)(p_2-2)$.

Case 2. For $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$, if $x, y \in B$. Similar to the previous case, the number of vertices closer to x than to y is 1, and vice versa. Thus, $n_x(xy|\overline{\Gamma}_{\mathbb{Z}_n})n_y(xy|\overline{\Gamma}_{\mathbb{Z}_n}) = 1$. Since B has p_1-1 elements, the Szeged index of this case is $\frac{1}{2}(p_1-1)(p_1-2)$.

Case 3. For $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$ if $x, y \in C$. The number of vertices closer to x than to y is 1 and vice versa. Hence, $n_x(xy|\overline{\Gamma}_{\mathbb{Z}_n})n_y(xy|\overline{\Gamma}_{\mathbb{Z}_n}) = 1$. Since $C = \mathbb{Z}_n \backslash (A, B, \{0\})$, then C has $n-p_2-p_1+1$ elements. Thus, the Szeged index of this case is $\frac{1}{2}(n-p_2-p_1)(n-p_2-p_1+1)$.

Case 4. For $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$, if $x \in A$ and $y \in C$. Based on Theorem 4.3, $n_x(xy|\overline{\Gamma}_{\mathbb{Z}_n})$ is 1, i.e., x itself, and $n_x(xy|\overline{\Gamma}_{\mathbb{Z}_n})$ is p_1, i.e., y and all elements in B. Since A has p_2-1 elements and C has $n-p_2-p_1+1$ elements, the Szeged index in this case is $p_1(p_2-1)(n-p_2-p_1+1)$.

Case 5. For $xy \in E(\overline{\Gamma}_{\mathbb{Z}_n})$, if $x \in B$ and $y \in C$. Based on Theorem 4.3, $n_x(xy|\overline{\Gamma}_{\mathbb{Z}_n})$ is 1, i.e., x itself, and $n_y(xy|\overline{\Gamma}_{\mathbb{Z}_n})$ is p_2, i.e., y and all elements in A. Since B has p_1-1 elements and C has $n-p_2-p_1+1$ elements, the Szeged index in this case is $p_2(p_1-1)(n-p_2-p_1+1)$.

By summing up the Szeged indices from each case, obtained: $Sz(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}((p_1-2)(p_1-1)+(p_2-2)(p_2-1)+(n-p_2-p_1)(n-p_2-p_1+1))+p_1(p_2-1)(n-p_2-p_1+1)+p_2(p_1-1)(n-p_2-p_1+1)$ □

3. Conclusion

The research reveals the characteristics of the Hyper-Wiener index and Szeged index of the non-coprime graph of the integer modulo n group. The obtained results are as follows:

1. $WW(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}(n-1)(n-2)$, if n is a prime power.
2. $Sz(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}(n-1)(n-2)$, if n is a prime power.
3. $WW(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}[(p_2-1)(p_2-2)+(p_1-1)(p_1-2)+(n-p_2-p_1)(n-p_2-p_1+1)]+(n-p_2-p_1+1)(p_1+p_2-2)+3(p_1-1)(p_2-1)$, if $n = p_1p_2$ is a multiplicaton of two distinct prime number.
4. $Sz(\overline{\Gamma}_{\mathbb{Z}_n}) = \frac{1}{2}((p_1-2)(p_1-1)+(p_2-2)(p_2-1)+(n-p_2-p_1)(n-p_2-p_1+1))+p_1(p_2-1)(n-p_2-p_1+1)+p_2(p_1-1)(n-p_2-p_1+1)$, if $n = p_1p_2$ is the product of two distinct prime numbers.

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