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Numerical Analysis of Mathematical Model for Diabetes Mellitus Disease by Using Adam-Bashfort Moulton Method

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ABSTRACT

Diabetes mellitus is a metabolic disorder characterized by elevated blood glucose levels, known as hyperglycemia. The objective of this study is to develop a mathematical model of diabetes mellitus. The model will be analyzed in terms of its equilibrium points using the Adam-Bashforth Moulton numerical method. The numerical method that used is a multistep method. The predictor step employs the Runge-Kutta method, while the corrector step uses the Adam-Bashforth Moulton method. The mathematical model of diabetes mellitus is categorized into two classes: uncomplicated diabetes mellitus and complicated diabetes mellitus. The resulting model identifies two equilibrium points: the endemic equilibrium point (complicated) and the disease-free equilibrium point (uncomplicated). The eigenvalues of these equilibrium points are positive real numbers and negative real numbers. Therefore, the stability of the system is found to be unstable and asymptotically stable, indicating that the population of individuals with uncomplicated diabetes mellitus will continue to rise, whereas the population with complications will not increase significantly over time.

Keywords: Diabetes Mellitus, Adam-Bashfort Moulton Method

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1. Introduction

Diabetes, technically called diabetes mellitus, is reffered to types of disorders in the metabolic processes of the human body in which controlling mechanism of sugar level in blood is disrupted (Shabestari et al, 2018). In addition, diabetes mellitus also is a glucose metabolism disease characterized by chronic hyperglycemia resulting from defects in insulin secretion, insulin action, or both (Gao, et al, 2017). In another definition, diabetes mellitus is a disease related to metabolism which is characterized by increased blood glucose levels or hyperglycemia (Poznyak et al, 2020). If not treated properly, it can cause various complications that affect a person's survival (Kaya et al., 2021). In order to make the explanation of diabetes mellitus easier to understand, it can be transformed into a model, specifically a mathematical model.

Mathematical models are part of mathematics that are often used in solving problems in life. Mathematical models are used to describe an event in the form of a mathematical equation and are analyzed to obtain a solution to the problem that occurs (Ndii, 2018). Differential equations are one of the mathematical models often used to solve modeling problems in various fields, including in the medical field. The solution to the problem can be solved analytically and numerically. One method for obtaining a solution to a mathematical model numerically is the Runge Kutta Method (Wijayanti et all., 2017).

Mathematical models in medicine are often used to model the spread of a disease. Diabetes mellitus is one of the diseases that can be modeled in the form of a mathematical model (Irwan, 2019). The implementation of mathematical modeling tools is an emerging trend (Barbolosi et al, 2016). The prevalence of diabetes mellitus has been steadily increasing both globally and in Indonesia. As one of the most common chronic diseases, diabetes poses a significant public health challenge (Ekawati, 2021). The rise in cases is attributed to various factors, including lifestyle changes, urbanization, and an aging population. In Indonesia, the growing number of individuals affected by diabetes is particularly concerning, as it not only impacts the health of the population but also places a substantial burden on the

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healthcare system (Mukhtar et al, 2020). This increasing trend underscores the urgent need for effective prevention, management, and treatment strategies to combat diabetes mellitus on a national and global scale.

The number of people with diabetes mellitus in the world has increased from year to year. Based on data from the International Diabetes Federation (IDF), in 2021, the number of adults living with diabetes worldwide is estimated to reach 537 million people. This figure is expected to increase to 642 million by 2040. To predict the increase in the population of people with diabetes, a mathematical model can be used. The clinical and epidemiological characteristics of diabetes mellitus and the quality of its therapy are the key prognostic dominant the determines the organizational aspects of the diabetic service (Dedov, et al., 2023).

A model is an abstraction that reduces a problem to its essential characteristics.Mathematical models are useful because they exemplify the mathematical core of situation without extraneous information (Akinsola and Oluyo, 2019). Mathematical models and numerical methods have been utilized as theoritical tools for years study fundamental elements of a wide range of healthcare (AlShubarji et al, 2023).

Therefore, in this study, a mathematical model for diabetes mellitus will be formed which is divided into two classes, namely diabetes mellitus without complications and diabetes mellitus with complications. The purpose of this study is to create a mathematical model of diabetes mellitus. Then the model will be analyzed based on its equilibrium point. Furthermore, it will be simulated using data that has been obtained using numerical methods. Based on this description, the author will study the mathematical model of diabetes mellitus disease and find its simulation by using Adam-Bashfort Moulton method.

2. Research Method

This study uses literature study and applied research. According to Irina (2017), applied mathematics is a research that has practical reasons, a desire to know, aims to be able to do something much better, more effective and efficient. In this study, a stability analysis will be carried out on the population of diabetes mellitus sufferers in a mathematical model.

In this research, a mathematical modeling approach is employed to analyze the dynamics of diabetes mellitus. The study utilizes a combination of analytical and numerical methods to investigate the behavior of the model under various conditions. Specifically, the Adam-Bashforth Moulton multistep method is applied for numerical simulations, with the Runge-Kutta method used in the predictor phase and the Adam-Bashforth Moulton method employed in the corrector phase. This approach allows for a detailed examination of the stability of equilibrium points and the long-term behavior of the system, providing insights into the progression of diabetes mellitus and the potential impact of different factors on the disease's trajectory.

Data sources are part of the supporting factors of a study. The data used in this study are secondary data, namely data from patients with diabetes mellitus. The data source was obtained from Mataram District Hospital, West Nusa Tenggara, Indonesia.

The research plan as follows:

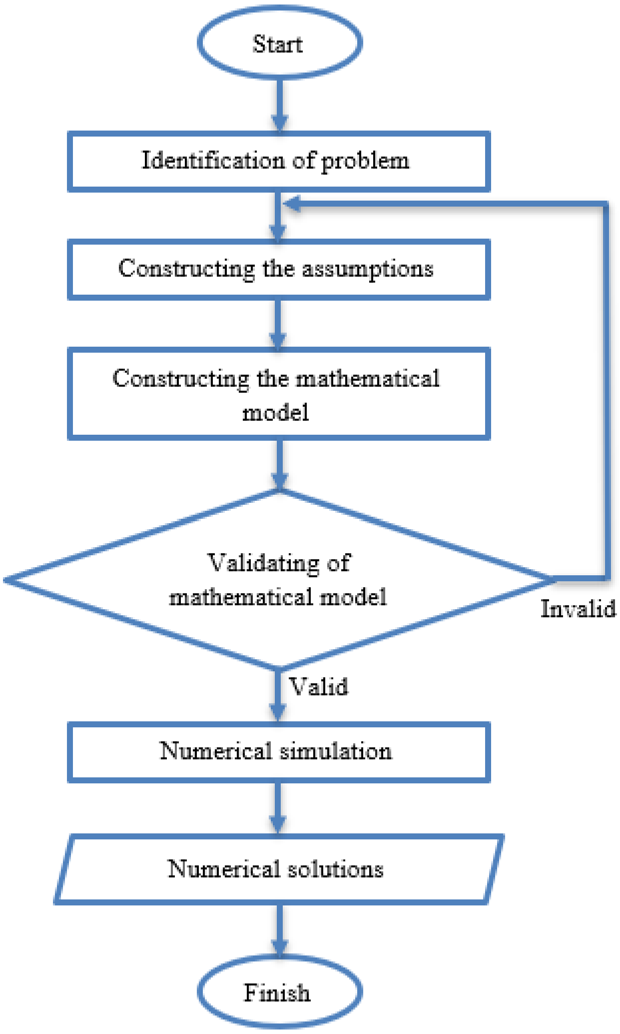

Figure. 1 Flowchart of research

- Formulating a real model means the characteristics of the problem related to diabetes mellitus.
- Making assumptions for the model based on observations.
- Formulating a mathematical problem (model).
- Simplifying the mathematical model with an appropriate method.
- Determining the equilibrium point (fixed).
- Linearization using the Jacobian matrix at a fixed point.
- Determining the eigenvalue (λ).
- Conducting numerical simulations to provide an overview based on the solutions obtained.
- Validating the mathematical model that has been obtained.

3. Constructing the Mathematical Model

Some facts is obtained related to diabetes mellitus are Diabetes mellitus is a metabolic disorder that is genetically and clinically heterogeneous with manifestations in the form of loss of carbohydrate tolerance, if it has fully developed clinically then diabetes mellitus is characterized by fasting and postprandial hyperglycemia, atherosclerosis and microangiopathic vascular disease. Furthermore, if diabetes mellitus is not treated properly it will cause complications that are fatal, one of which is death. Diabetes complications can be prevented, delayed or slowed down by controlling blood sugar levels. By carrying out therapy and administering drugs, blood sugar levels can be controlled so that the survival of diabetes mellitus sufferers is longer.

Based on the facts obtained, in this study, assumptions will be made so that the model formed can be limited and clarified. The assumptions made in forming the model are as follows:

- The population of sufferers is considered constant.
- Natural deaths occur in each population class.
- The number of new cases of diabetes sufferers (Incidence) is included in the population class of diabetes sufferers without complications.
- Temporary healing occurs due to controlling blood sugar levels.
- Total healing occurs from complications to no complications.
- Death occurs due to complications.

Based on the assumptions that have been made, the variables in the mathematical model in this study are the number of people with diabetes mellitus without complications (A), the number of people with diabetes mellitus with complications (B), and the total number of people with diabetes mellitus (N). The mathematical model of the diabetes mellitus population at time t can be described in the following Figure 2:

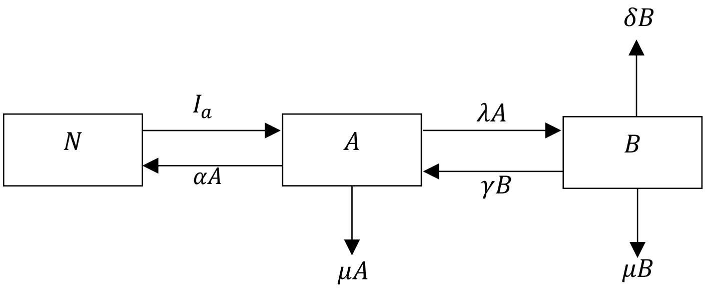

Figure. 2 Compartment diagram of population model for Diabetes Mellitus sufferer

According to the Figure 2, the mathematical model for Diabetes Mellitus sufferer can genarate to be

$$\frac{dA}{dt} = I_a + \gamma B - \lambda A - \mu A - \alpha A$$
$$\frac{dA}{dt} = I_a - (\lambda + \mu + \alpha)A + \gamma B \quad (1)$$

$$\frac{dB}{dt} = \lambda A - \gamma B - \mu B - \delta B \quad (2)$$
$$\frac{dB}{dt} = \lambda A - (\gamma + \mu + \delta)B$$

In Equation 1 presents the rate of change of the population of people with diabetes mellitus without complications over time$\left(\frac{dA}{dt}\right)$ increases due to the presence of individuals diagnosed with diabetes mellitus I_a and the recovery of individuals from complications to no complications (γB). Then decreases due to the presence of individuals experiencing complications (λA), natural deaths (μA), and individuals categorized as temporarily cured (αA).

In Equation 2 presents the rate of change of the population of diabetes mellitus sufferers with complications over time $\left(\frac{dB}{dt}\right)$ increases due to the presence of individuals experiencing complications (λA). Then decreases due to natural deaths (μB), deaths due to complications (δB), and individuals who recover from complications (γC). Furthermore, the total population of diabetes mellitus sufferers is $N = A + B$.

Based on the mathematical model Equation 1 and Equation 2, it can be seen that $N = A + B$, so $N - B = A$ is obtained. It is assumed that $\sigma = \lambda + \mu + \alpha$ and $\theta = \gamma + \mu + \delta$ to make it easier to analyze the model, so that the differential equation system of Equation 1 and Equation 2 can be rewritten as follows:

a) $\frac{dA}{dt} = I_a - (\lambda + \mu + g)A + \gamma B$

$$\frac{dA}{dt} = I_a - \sigma A + \gamma B \quad (3)$$

b) $\frac{dB}{dt} = \lambda A - (\gamma + \mu + \delta)B$

$$\frac{dB}{dt} = \lambda(N - B) - \theta B$$
$$\frac{dB}{dt} = \lambda N - \lambda B - \theta B$$
$$\frac{dB}{dt} = \lambda N - (\lambda + \theta)B \quad (4)$$

c) $\frac{dN}{dt} = \frac{dA}{dt} + \frac{dB}{dt}$

$$\frac{dN}{dt} = (I_a - (\lambda + \mu + g)A + \gamma B) + (\lambda A - (\gamma + \mu + \delta)B)$$
$$\frac{dN}{dt} = I_a - (\lambda + \mu + g)A + \gamma B + \lambda A - (\gamma + \mu + \delta)B$$
$$\frac{dN}{dt} = I_a - \lambda A - \mu A - \alpha A + \gamma B + \lambda A - \gamma B - \mu B - \delta B$$
$$\frac{dN}{dt} = I_a - \mu A - \alpha A - \mu B - \delta B$$
$$\frac{dN}{dt} = I_a - (A + B)\mu - \alpha A - \delta B$$
$$\frac{dN}{dt} = I_a - \mu N - \alpha A - \delta B \quad (5)$$

If β is the controlling coefficient used to define λ, then the assumed proportion of increasing diabetes mellitus complications is as follows:

$$\lambda = \beta \frac{C}{N} \tag{6}$$

By substituting Equation 6 into Equation 4, we will obtain a new differential Equation 4 as follows:

$$\frac{dB}{dt} = (\beta - \theta)B - \beta \frac{B^2}{N}.$$

Then, the population model of diabetes mellitus sufferers can be written as system as follows:

$$\begin{aligned} \frac{dA}{dt} &= I_d - \sigma A + \gamma B \\ \frac{dB}{dt} &= (\beta - \theta)B - \beta \frac{B^2}{N} \\ \frac{dN}{dt} &= I_d - \mu N - \alpha A - \delta B \end{aligned} \tag{7}$$

where

I_a: incidence of diabetes mellitus
λ : proportion of diabetes mellitus sufferers increasing to the level of complications
μ : natural death rate
α : blood glucose level
γ : complication recovery rate
δ : death rate of sufferers due to complications
and $I_d, \lambda, \mu, \alpha, \gamma, \delta \geq 0$.

4. Results

4.1. Equilibrium Point

The equilibrium point is obtained by setting the right-hand side of the system of Equations 7 to zero $\left(\frac{dA}{dt} = 0,\ \frac{dB}{dt} = 0,\ \frac{dN}{dt} = 0\right)$. This results in the system of equations 8 as follows:

$$\begin{aligned} I_a - \sigma A + \gamma B &= 0 \\ (\beta - \theta)B - \beta \frac{B^2}{N} &= 0 \\ I_a - \mu N - \alpha A - \delta B &= 0 \end{aligned} \tag{8}$$

Based on $(\beta - \theta)B - \beta \frac{B^2}{N} = 0$, we have $\hat{B} = 0$ or $B = \frac{(\beta-\theta)N}{\beta}$, then there is two equilibrium points of its condition.

a) Equilibrium point (T_1)

Determining the equilibrium point is done by assuming $B = 0$, which means there is no population experiencing complications. Then, the value $B = 0$ is substituted into $I_a - \sigma A + \gamma B = 0$, the results is

$$I_a - \sigma A + \gamma B = 0$$
$$I_a - \sigma A + \gamma(0) = 0$$
$$\sigma A = I_a$$

Next, substitute $B = 0$ and $\hat{A} = \frac{I_a}{\sigma}$ into $I_a - \mu N - \alpha A - \delta B = 0$, then obtained

$$I_d - \mu N - \alpha A - \delta B = 0$$
$$\mu N = I_a - \alpha\left(\frac{I_a}{\sigma}\right) - \delta(0)$$
$$\mu N = I_a - \frac{\alpha I_a}{\sigma}$$
$$\mu N = \frac{I_a \sigma - \alpha I_a}{\sigma}$$
$$\mu N = \frac{I_a(\sigma - \alpha)}{\sigma}$$
$$\hat{N} = \frac{I_a(\sigma - \alpha)}{\sigma\mu}$$

Therefore, we are obtained the first equilibrium point as follows:

$$T_1 = (\hat{A}, \hat{B}, \hat{N}) = \left(\frac{I_a}{\sigma}, 0, \frac{I_a(\sigma-\alpha)}{\sigma\mu}\right).$$

b) Equilibrium point (T_2)

The equilibrium point is determined by assuming $B = \frac{(\beta-\theta)N}{\beta}$, which means there is a population experiencing complications. Then, the value $B = \frac{(\beta-\theta)N}{\beta}$ is substituted into $I_a - \mu N - \alpha A - \delta B = 0$, resulting in:

$$I_a - \sigma A + \gamma B = 0$$
$$I_a - \sigma A + \gamma\left(\frac{(\beta - \theta)N}{\beta}\right) = 0$$
$$I_a - \sigma A + \gamma\left(\frac{(\beta - \theta)N}{\beta}\right) = 0$$
$$I_a - \sigma A + \frac{\gamma(\beta - \theta)N}{\beta} = 0$$
$$I_a - \sigma A + \frac{\gamma\beta N - \gamma\theta N}{\beta} = 0$$
$$\sigma A = I_a + \frac{\gamma\beta N - \gamma\theta N}{\beta}$$
$$\sigma\beta A = \beta I_a + \gamma\beta N - \gamma\theta N$$
$$A = \frac{\beta I_a + \gamma\beta N - \gamma\theta N}{\sigma\beta}.$$

Substitute $B = \frac{(\beta-\theta)N}{\beta}$ and $A = \frac{\beta I_a+\gamma\beta N-\gamma\theta N}{\sigma\beta}$ into $I_a - \mu N - \alpha A - \delta B = 0$, the we have $I_a - \mu N - \alpha A - \delta B = 0$

$$I_a - \mu N - \alpha\left(\frac{\beta I_a+\gamma\beta N-\gamma\theta N}{\sigma\beta}\right) - \delta\left(\frac{(\beta-\theta)N}{\beta}\right) = 0$$

$$\sigma\beta I_a - \sigma\beta\mu N - \alpha(\beta I_a + \gamma\beta N - \gamma\theta N) - \sigma\delta(\beta N - \theta N) = 0$$

$$\sigma\beta I_a - \sigma\beta\mu N - \alpha\beta I_a - \alpha\gamma\beta N + \alpha\gamma\theta N - \sigma\delta\beta N + \sigma\delta\theta N = 0$$

$$\sigma\beta I_d - \alpha\beta I_d - N(\sigma\beta\mu + \alpha\gamma\beta - \alpha\gamma\theta + \sigma\delta\beta - \sigma\delta\theta) = 0$$

$$N(\sigma\beta\mu + \alpha\gamma\beta - \alpha\gamma\theta + \sigma\delta\beta - \sigma\delta\theta) = \sigma\beta I_d - \alpha\beta I_d$$

$$\widehat{N} = \frac{\sigma\beta I_d - \alpha\beta I_d}{\sigma\beta\mu + \alpha\gamma\beta - \alpha\gamma\theta + \sigma\delta\beta - \sigma\delta\theta}$$

$$\widehat{N} = \frac{\beta I_d(\sigma-\alpha)}{\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta)}.$$

By substitute $\widehat{N} = \frac{\beta I_d(\sigma-\alpha)}{\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta)}$ into $C = \frac{(\beta-\theta)N}{\beta}$ is obtained

$$\hat{A} = \frac{\beta I_a + \gamma\beta\left(\frac{\beta I_d(\sigma-\alpha)}{\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta)}\right) - \gamma\theta\left(\frac{\beta I_d(\sigma-\alpha)}{\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta)}\right)}{\sigma\beta}$$

$$\hat{A} = \frac{I_a(\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta))+\gamma I_d(\beta-\theta)(\sigma-\alpha)}{\sigma(\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta))}$$

Therefore, we are obtained the second equilibrium point $T_2 = (\hat{A}, \hat{B}, \widehat{N})$ as follows

$$\hat{A} = \frac{I_a(\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta))+\gamma I_a(\beta-\theta)(\sigma-\alpha)}{\sigma(\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta))}$$

$$\hat{B} = \frac{I_a(\beta-\theta)(\sigma-\alpha)}{\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta)}$$

$$\widehat{N} = \frac{\beta I_a(\sigma-\alpha)}{\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta)}.$$

4.2. *Eigen value of system*

Before obtaining the eigenvalues, the process of linearization must be performed. The linearization process is carried out to determine the stability of the model's equilibrium points. Linearization of the system of equations is done using the Jacobian matrix (J). The form of the Jacobian matrix based on system 7 is as follows:

$$J = \begin{bmatrix} \frac{\partial\left(\frac{\partial D}{\partial t}\right)}{\partial D} & \frac{\partial\left(\frac{\partial D}{\partial t}\right)}{\partial C} & \frac{\partial\left(\frac{\partial D}{\partial t}\right)}{\partial N} \\ \frac{\partial\left(\frac{\partial C}{\partial t}\right)}{\partial D} & \frac{\partial\left(\frac{\partial C}{\partial t}\right)}{\partial C} & \frac{\partial\left(\frac{\partial C}{\partial t}\right)}{\partial N} \\ \frac{\partial\left(\frac{\partial N}{\partial t}\right)}{\partial D} & \frac{\partial\left(\frac{\partial N}{\partial t}\right)}{\partial C} & \frac{\partial\left(\frac{\partial N}{\partial t}\right)}{\partial N} \end{bmatrix}$$

$$J = \begin{bmatrix} -\sigma & \gamma & 0 \\ 0 & (\beta-\theta) - 2\beta\frac{B}{N} & \beta\frac{B^2}{N^2} \\ -\alpha & -\delta & -\mu \end{bmatrix}$$

The stability of the linearized system can be examined through the eigenvalues of the Jacobian matrix that has been obtained. There are two equilibrium points as follows:

- The equilibrium point $T_1 = (\hat{A}, \hat{B}, \widehat{N}) = \left(\frac{I_a}{\sigma}, 0, \frac{I_a(\sigma-\alpha)}{\sigma\mu}\right)$ is substituted into the Jacobian matrix J and is obtained:

$$J_1 = \begin{bmatrix} -\sigma & \gamma & 0 \\ 0 & (\beta-\theta) & 0 \\ -\alpha & -\delta & -\mu \end{bmatrix}$$

Next, based on J_1, the characteristic equation can be obtained by $det(\lambda I - J_1) = 0$, where I is identity matrix.

$$det(\lambda I - J_1) = 0$$

$$det\left(\lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -\sigma & \gamma & 0 \\ 0 & (\beta-\theta) & 0 \\ -\alpha & -\delta & -\mu \end{bmatrix}\right) = 0$$

$$(\lambda+\mu)\big((\lambda+\sigma)(\lambda-\beta+\theta) - 0\big) = 0$$

$$(\lambda+\mu)(\lambda+\sigma)(\lambda-\beta+\theta) = 0.$$

Thus, the eigenvalues obtained $\lambda_1 = -\mu$, $\lambda_2 = -\sigma$, $dan\ \lambda_3 = \beta - \theta.$ Since $\beta - \theta > 0$, the resulting eigenvalues include both positive and negative real numbers. This indicates that the system of equations 7 around the equilibrium point T_1 is unstable.

- The equilibrium point $T_2 = (\hat{A}, \hat{B}, \widehat{N})$ is substituted into Jacobian matrix, then we are obtained:

$$T_2 = (\hat{A}, \hat{B}, \widehat{N})$$

$$J_2 = \begin{bmatrix} -\sigma & \gamma & 0 \\ 0 & -(\beta-\theta) & \frac{(\beta-\theta)^2}{\beta} \\ -\alpha & -\delta & -\mu \end{bmatrix}.$$

Based on J_2, the characteristic equation can be obtained by $det(\lambda I - J_2) = 0$, where I is identity matrix.

$$det(\lambda I - J_2) = 0$$

$$det\left(\lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -\sigma & \gamma & 0 \\ 0 & -(\beta-\theta) & \frac{(\beta-\theta)^2}{\beta} \\ -\alpha & -\delta & -\mu \end{bmatrix}\right) = 0$$

$$\begin{vmatrix} \lambda+\sigma & -\gamma & 0 \\ 0 & \lambda + (\beta-\theta) & -\frac{(\beta-\theta)^2}{\beta} \\ \alpha & \delta & \lambda+\mu \end{vmatrix} = 0$$

Recall that $\theta = \gamma + \mu + \delta$, as different values of γ will be used. The eigenvalues will be obtained after substituting the parameter values into d $det(\lambda I - J_2)$. The resulting eigenvalues will be λ_1, λ_2 and λ_3. Based on the eigenvalues, the possible scenarios are:

- If λ_1, λ_2 and λ_3 are negative real numbers, this indicates that the system of equations 7 around the equilibrium point T_2 is asymptotically stable.
- If λ_1, λ_2 and λ_3 include both positive and negative real numbers, this indicates that the system of equations 7 around the equilibrium point T_2 is unstable.

4.3 Numerical Simulation

The simulation model is provided to give an overview and validation of the developed model. The simulation uses data from diabetes mellitus patients in 2016 at the Mataram General Hospital. It is assumed that $I_a = 1280$. According to data from the hospital, the total number of diabetes mellitus patients is 1280, with 1000 patients suffering from complications and 280 patients without complications. Therefore, the simulation is conducted by assigning values to each parameter.

Table 1 – Parameter Value.

Parameter	The Value of yr^{-1}
μ	0.03
δ	0.06
α	0.05
γ	0.09 or 0
β	1
λ	0.90

4.4 The Condition When Patients Recover from Complications

The simulation of the diabetic population model under this condition uses parameter values from Table 1. In this part, the recovery rate from complications (γ) is 0.09, resulting in $\theta = 0.16$. Substituting these values into the system of equations 7, we obtain the following:

$$\frac{dA}{dt} = I_a - 0{,}91A + 0{,}08B$$
$$\frac{dB}{dt} = 0{,}85B - \frac{B^2}{N} \qquad (9)$$
$$\frac{dN}{dt} = 1170 - 0{,}02N - 0{,}05B - 0{,}04A.$$

The stability of the system of equations 9 can be analyzed through the eigenvalues of its Jacobian matrix. At the equilibrium point $T_1(1285{,}71, 0, 55928{,}57)$, the eigenvalues are $\lambda_1 = -0{,}02$, $\lambda_2 = -0{,}91$, and $\lambda_3 = 0{,}85$. It can be observed that the eigenvalues consist of both positive and negative real numbers, indicating that the system of equations 9 around the equilibrium point T_1 is unstable.

Next, the stability of the system of equations 9 around the equilibrium point $T_2(2562{,}04, 14518{,}2, 17080{,}3)$ will be investigated. At the equilibrium point T_2, the eigenvalues are $\lambda_1 = -0{,}698313$, $\lambda_2 = -0{,}931029$, and $\lambda_3 = -0{,}777914$ Since all the eigenvalues are negative real numbers, it can be concluded that the system of equations 9 around the equilibrium point T_2 is asymptotically stable.

The stability of the system of equations 9 can also be analyzed based on its phase portrait as follows:

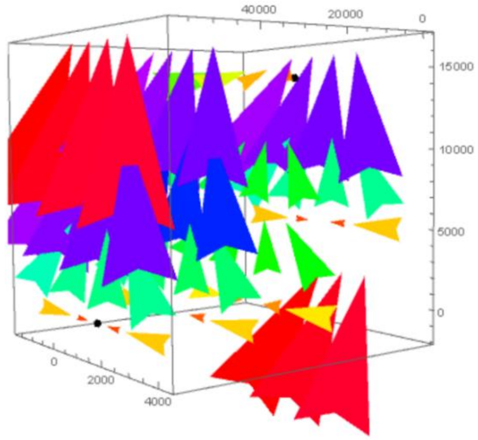

Figure. 2 – Phase portrait of the population of Diabetes Mellitus patients recovering from complications

Using the Adam-Bashfort Moulton method as a corrector for simulation, we need Runge-Kutta method as predictor for its simulation first (Suryanto, 2017). The rate of change in the population of diabetes mellitus patients can be determined, as shown in the following graph:

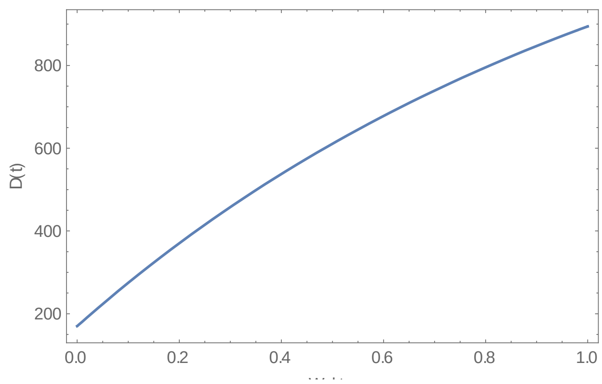

Figure. 3 – Graph of the rate of change in the total population of Diabetes Mellitus patients without complications

Based on Figure 3, it can be observed that the population of diabetes mellitus patients without complications is steadily increasing. The rate of change in the population of diabetes mellitus patients over time, under the condition where some patients recover from complications, shows a continuous increase over time (years). The fundamental difference between the two graphs is that the number of patients in the condition where no individuals recover from complications is lower than in the condition where individuals do recover from complications.

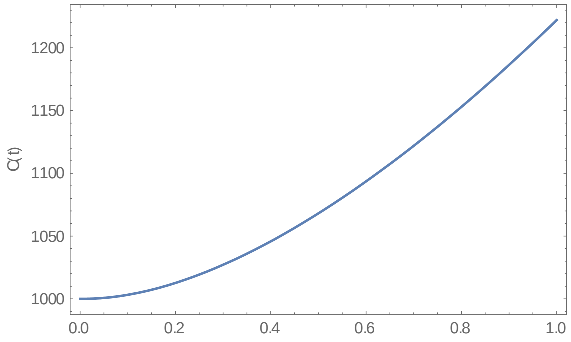

Figure. 4 – Graph of the rate of change in the total population of Diabetes Mellitus patients with complications

Based on Figure 4, it can be observed that the population of diabetes mellitus patients with complications is also steadily increasing. However, unlike the rate of the population of diabetes mellitus (DM) patients without complications, these two graphs illustrate that the number of DM patients with complications is higher in the condition where no patients recover compared to the condition where some patients do recover from complications. This is due to the transition of patients from having complications to being without complications. Therefore, it can be said that the survival rate of DM patients depends on the number of individuals who recover. The likelihood of death due to DM can be minimized by providing various treatments to ensure that DM patients who experience complications can be cured of them.

4.5 The Condition When Patients Do Not Complications

The simulation of the diabetic population model under this condition uses the parameter values from Table 1. In this scenario, the recovery rate from complications (γ) is 0, resulting in $\theta = 0{,}07$. Substituting these values into the system of equations 7 yields the following:

$$\frac{dA}{dt} = I_a - 0{,}91A + 0B$$
$$\frac{dB}{dt} = 0{,}93B - \frac{B^2}{N}(5.7) \qquad (10)$$
$$\frac{dN}{dt} = 1170 - 0{,}02N - 0{,}05B - 0{,}04A$$

The equilibrium point is $T_1(1285{,}71{,}0{,}55928{,}57)$ and the stability of the system of equations 10 can be analyzed through the eigenvalues of its Jacobian matrix. At the equilibrium point $T_1(1285{,}71\,,0\,,55928{,}57)$, the eigenvalues are $\lambda_1 = -0{,}02$, $\lambda_2 = -0{,}91$, and $\lambda_3 = 0{,}93$. Since the eigenvalues consist of both positive and negative real numbers, the system of equations 10 around the equilibrium point T_1 is unstable.

Next, the stability of the system of equations 10 will be investigated around the equilibrium point $T_2(2596{,}48, 14909{,}9, 16032{,}2)$. At the equilibrium point T_2, T_2. Since all the eigenvalues are negative real numbers, the system of equations 10 around the equilibrium point T_2 is asymptotically stable.

The stability of the system of equations 10 can also be analyzed based on its phase portrait as follows:

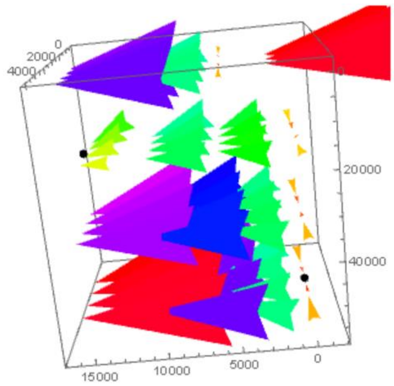

Figure. 5 – Phase portrait of the population of Diabetes Mellitus with complications

Using the Adam-Bashfort Moulton method, the rate of change in the population of diabetes mellitus patients can be determined. The rate of change in the population of diabetes mellitus patients over time, under the condition that no patients recover from complications, shows an increasing trend each year.

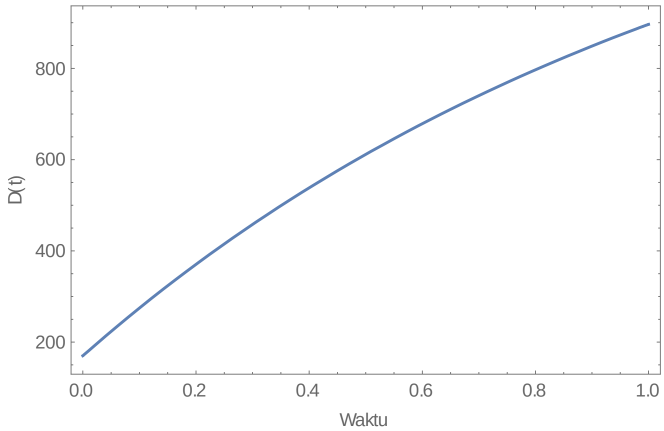

Figure. 6 – Graph of the rate of change in the population of Diabetes Mellitus patients without complications

Based on Figure 6, it can be observed that the population of diabetes mellitus patients without complications is steadily increasing.

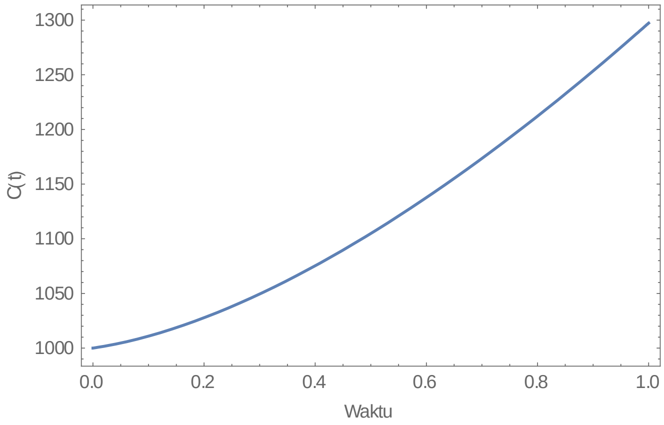

Figure. 7 – Graph of the rate of change in the total population of Diabetes Mellitus patients with complications

Based on Figure 7, it can be observed that the population of diabetes mellitus patients without complications is steadily increasing.

Based on the stability values of both conditions, it can be determined that the value of γ does not affect the stability of the system. However, an increase in the value of γ can reduce the population of diabetes mellitus patients with complications.

5. Discussions

In this study, to achieve the stated objectives, data related to diabetes mellitus (DM) with and without complications were first collected. The collected data were then used to establish assumptions that would be utilized to formulate a mathematical model, resulting in the model for DM patients as shown in Equation 1.

The first model in Equation 1 shows that the rate of population growth in DM patients is influenced by the incidence of DM, meaning the number of individuals diagnosed with DM is assumed to be constant. The presence of individuals who recover from complications also influences this growth, in addition to the natural death rate, the proportion of DM progressing to complications, and the rate at which individuals successfully normalize their blood sugar levels, all of which can reduce the growth rate of DM patients without complications.

The second model in Equation 1 indicates that the population without complications is influenced by the proportion of individuals who develop complications. The more individuals who experience complications, the greater the population of DM patients with complications will grow, indicating that the death rate of individuals will also increase over time.

The resulting model is a set of differential equations, which are then solved by forming a new model. In the system of Equations 2, it can be seen that a new model is formed, where this system creates a nonlinear system of equations. The equilibrium points and their stability analysis are then determined from this model.

The analysis results yield two equilibrium points: the endemic (complications) equilibrium and the disease-free (without complications) equilibrium. The disease-free equilibrium points are given successively as $\left(\hat{A},\hat{B},\hat{N}\right) = \left(\frac{I_a}{\sigma}, 0, \frac{I_a(\sigma-\alpha)}{\sigma\mu}\right)$, $\hat{A} \to \frac{I_a}{\sigma}$ means that as time progresses, the total population will approach the equilibrium point $A.\hat{B} \to 0$means that as time progresses, the number of individuals entering class B will diminish. $\hat{N} \to \frac{I_a(\sigma-\alpha)}{\sigma\mu}$ means that as time progresses, the total population will approach the equilibrium point N. The endemic (complications) equilibrium points are successively given as $\left(\hat{A},\hat{B},\hat{N}\right) =$

$$\left(\begin{array}{c}\frac{I_d\left(\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta)\right)+\gamma I_d(\beta-\theta)(\sigma-\delta)}{\sigma\left(\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta)\right)}, \\ \frac{I_d(\beta-\theta)(\sigma-\delta)}{\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta)}, \\ \frac{\beta I_d(\sigma-\delta)}{\sigma\beta\mu+\alpha\gamma(\beta-\theta)+\sigma\delta(\beta-\theta)}\end{array}\right)$$

The stability analysis of the linearized system can be examined through the eigenvalues of the Jacobian matrix that was obtained. From the previous analysis, the eigenvalues of the first equilibrium point (T_1) are found to be distinct real numbers, one positive and one negative, indicating that the stability of the system is unstable, meaning the population of DM patients will continually increase. Meanwhile, the eigenvalues of the second equilibrium point (T_2) are found to be negative real numbers, indicating that the stability of the system is asymptotically stable, meaning the population of DM patients will not increase significantly over time.

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