Volume. 7 No 2 December (2024)

e-ISSN: **2615-3270**
p-ISSN: **2615-3599**

Eigen Mathematics Journal

Homepage jurnal: http://eigen.unram.ac.id/ index.php/eigen

Stock Portfolio Optimization Using Single Index Model (SIM) with Exponentially Weighted Moving Average (EWMA) Approach

Ainul Mutmainna [a]* , Nurwahidah [b] , Sri Dewi Anugrawati [c]

[a.] Department of Mathematics, Universitas Islam Negeri Alauddin Makassar. Email: ainulmutmainna29@gmail.com
[b.] Department of Mathematics, Universitas Islam Negeri Alauddin Makassar. Email: nurwahidah.abidin@uin-alauddin.ac.id
[c.] Department of Mathematics, Universitas Islam Negeri Alauddin Makassar, Email: sridewi.anugrawati@uin-alauddin.ac.id

ABSTRACT

The optimal portfolio is a combination of various assets with the aim of reducing investment risk through diversification. This study aims to conduct stock selection using K-Means Clustering and the formation of an optimal stock portfolio from the application of Single Index Model the amount of investment risk in the portfolio using the Exponentially Weighted Moving Average approach, and the amount of portfolio performance. The analysis results show that there are 5 portfolios formed. The best portfolio that can be chosen by investors depends on the investor's risk tolerance. Investors with low risk tolerance can choose Portfolio 3 consisting of ICBP and MIKA stocks with an expected return of 0.01343 and a risk of 0.00714 and a VaR of IDR 2,633,286.63. Investors with moderate risk tolerance can choose Portfolio 1 which consists of ICBP, MIKA, ACES, INCO, ITMG, MAPI, TPIA, AKRA, and MDKA stocks with an expected return of 0.022047, risk of 0.01277 and VaR of IDR 3,083,287.87. Investors with high risk tolerance can choose Portfolio 2 which consists of MIKA, TPIA, and MDKA stocks with an expected return of 0.02504 and a risk of 0.01471 and a VaR of IDR 3,553,167.10.

Keywords: Portfolio, Single Index Model (SIM), Exponentially Weighted Moving Average (EWMA), K-Means Clustering

Submitted: 15-10- 2024; Accepted: 12-12- 2024; Doi: https://doi.org/10.29303/emj.v7i2.247

1. Introduction

Investment is a form of capital investment of a number of funds or other assets made at this time, with the aim of obtaining a number of benefits in the future (Tandelilin, 2010). Stock investment is a popular choice among investors because the frequency of stock trading is higher than the frequency of other investments in the capital market, besides that investment in stocks also has good opportunities in the long term. The basic thing that is considered in stock investment is to calculate the profit (return) and the level of risk (risk). Efforts in calculating return and risk considerations in stock investment can be done by implementing a diversification strategy through portfolio formation.

The objective of forming an optimal portfolio is to obtain an optimal combination of assets from several stocks. To form a portfolio, it is necessary to consider the allocation of funds to ensure that a combination of two or more types of stocks can generate optimal profits.

Portfolio formation will help investors in making decisions to choose which portfolio is optimal and has the expected level of return and with the minimum possible risk. Therefore, before forming an optimal portfolio an investor needs to determine a portfolio derived from efficient stocks. Optimal portfolio formation can be done using the Single Index Model (SIM) method, a fairly accurate approach developed by Sharpe (1963). The SIM calculation method is used to form an optimal portfolio and eliminate inefficient stocks based on their risk and return ratios (Setiawan, 2017).

Previous research by Septyanto and Kertopati (2014) showed that the Single Index Model method is the most efficient method for forming portfolios, obtained the largest expected return calculation of 0.596% with the smallest risk

* Corresponding author.
e-mail address: ainulmutmainna29@gmail.com

of 0.0264. Aunillah and Wahyudi (2022) who analyzed the optimal portfolio of CAPM and Single Index Model methods on IDX30 companies, obtained the most optimal portfolio resulting from the calculation of the Single Index Model with a portfolio return rate of 1.38% and a risk of 0.57%. Another study by Rimbawan, et al (2023) using the Single Index Model and Stochastic Dominance in the formation of the optimal portfolio of IDX stocks, obtained portfolio performance with the SIM method resulting in a better comparison of return and risk compared to the Stochastic Dominance method.

The stock portfolio optimization problem is basically a dynamic problem related to stochastic fluctuations in stock prices. The decline in share prices is a risk accepted by shareholders and parties who are receiving guarantees for the shares concerned and is known as market risk in the capital market. Investors can estimate the risk of a decrease in return value using Value at Risk with a certain level of confidence and within a certain period of time. The indicator used in the calculation of Value at Risk is volatility (σ). Stock movements are often characterized by volatility that is not constant or heteroscedastic. To overcome this, one common approach is the Exponentially Weighted Moving Average (EWMA) method (Hull and White, 1998).

Muis's research (2018) on predicting volatility in stocks listed in the Jakarta Islamic Index using the EWMA method, it was found that the EWMA method was valid enough to be used to calculate the maximum potential loss from investing in stocks listed in the JII. Konan, et al (2022) who examined the calculation of stock portfolio risk using the Exponentially Weighted Moving Average and Semi Variance methods found that the potential loss caused by diversified stock investment is smaller than the potential loss caused by non-diversified stock investment. Another study by Maheresmi (2023) comparing stock volatility using the EWMA, GARCH, and EGARCH methods in forming the optimal portfolio, obtained better risk and return using the EWMA method with an expected return portfolio of 0.000653755.

In determining investment decisions, stock combinations are indispensable in obtaining an optimal portfolio. Determining the combination of stocks helps investors in creating an optimal portfolio with diversified risk. In other words, an individual's investment strategy depends on how much risk tolerance the investor has to achieve the expected returns. Therefore, to avoid risk, investors choose some stocks with the smallest variance in their investment portfolio (Siregar & Pangruruk, 2021). Cluster analysis is used as a first step in selecting several stocks to form a portfolio. By grouping stocks, investors can build a more diversified portfolio. Each cluster represents different characteristics, so the risk can be spread more evenly. In this research, the clustering method used is K-means Clustering which is an unsupervised learning algorithm for grouping data based on the similarity of data characteristics. The K-means method is quite simple and efficient in clustering data.

Based on this description, this study applies the application of the Single Index Model (SIM) to optimize the stock portfolio with the Exponentially Weighted Moving Average (EWMA) approach through stock selection using K-Means Clustering so as to obtain an optimal stock portfolio.

2. Literature Review

2.1 Investment

Investment is the allocation of funds or other resources made at this time, with the aim of obtaining a number of benefits in the future (Tandelilin, 2010). Investment is the investment of capital in the company, with the aim that the wealth of a corporation or company increases.

The most basic understanding for an investor is that risk always follows return (risk and return trade off). The relationship between return is directly proportional to the level of risk. This means that the higher the expected level of return, the higher the level of investment risk obtained.

2.2 Shares

Shares are proof of capital or fund ownership in a company. Shares can also be defined as securities that are traded in the capital market. Stocks are the most popular product in the financial market and are used as an investment alternative because they are more profitable than other investments such as mutual funds and bonds (Azizah, 2021).

2.3 Return on Investment

In investment management, getting profit from the investment made is the hope of every investor. The profit obtained from an investment policy is called return. The return from investment can be in the form of realized return, which is the return that has occurred and is calculated based on historical data and expected return, which is the return expected by investors in the future.

The return of a stock can be calculated using the following formula:

$$R_t = \frac{S_t - S_{t-1}}{S_{t-1}} \quad (1)$$

where R_t is the return in period *t*, S_t is the stock price in period *t* and S_{t-1} is the stock price in period *t*-1.

In addition, mathematically, the expected return can be calculated using the following formula:

$$E(R_i) = \frac{\sum_{t=1}^{n} R_{it}}{n} \quad (2)$$

where $E(R_i)$ is the expected return of a security, R_{it} is the actual *return* in *i* at time *t* and n is the number of observations.

2.4 Risk in Investment

Risk is the uncertainty in investing to generate the expected rate of return in a certain period or in other words, risk is the degree of deviation (standard deviation) from expected results. Investment risk is obtained by the following equation:

$$\sigma = \sqrt{\sum_{i=1}^{n} \frac{\left(R_i - E(R_i)\right)^2}{n-1}} \quad (3)$$

2.5 *K-Means Clustering*

K-Means algorithm is a distance-based clustering algorithm, the smaller the distance between samples, the higher the similarity. The K-Means algorithm will divide samples into k categories based on a distance function using the Euclidean distance method (Shao et al., 2022). To calculate the distance of each data to the center using the Euclidean formula as follows:

$$D(i,j) = \sqrt{\left(X_{1i} - X_{1j}\right)^2 + \left(X_{2i} - X_{2j}\right)^2 + \cdots + \left(X_{ki} - X_{kj}\right)^2} \quad (4)$$

To calculate the cluster center in each cluster in each iteration, the following formula is used:

$$m_k = \frac{1}{|M_k|} \sum_{x_i \in M_k} X_i \quad (5)$$

2.6 Single Index Model

The Single Index Model links the calculation of asset returns to the market index with the assumption that securities will be correlated only if they have the same response to market changes (Tandelilin, 2017). The single index model is a very simple model for the return of assets, which is mathematically written as follows (Adnyana, 2020):

$$R_i = \alpha_i + \beta_i R_m + e_i \quad (6)$$

Here, R_i is the return from the i-th asset and R_m is the return from a market index, α_i and β_i are constants to be determined from the historical data and e_i is the residual error.

SIM portfolio analysis consists of calculating the expected return and portfolio risk. Mathematically, the expected return of the portfolio can be expressed as follows (Chakrabarty & Kanaujiya, 2023):

$$\begin{aligned} R_p &= \alpha_p + \beta_p R_m + e_p \\ E(R_p) &= E(\alpha_p + \beta_p R_m + e_p \\ &= E[\alpha_p] + E[\beta_p R_m] + E[e_p] \\ &= \alpha_p + \beta_p E(R_m), \text{ Karena } E[e_p] = 0 \end{aligned} \quad (7)$$

The risk of a well-diversified portfolio consists only of the systematic risk element which is mathematically expressed as follows (Chakrabarty & Kanaujiya, 2023):

$$\begin{aligned} Var(R_p) &= Var(\alpha_p + \beta_p R_m + e_p) \\ &= Var(\alpha_p) + Var(\beta_p R_m) + Var(e_p) + \\ &\quad Cov(\alpha_p, \beta_p R_m) + Cov(\alpha_p, e_p) + \\ &\quad Cov(\beta_p R_m, e_p) \\ &= 0 + \beta_p^2 Var(R_m) + Var(e_p) + 0 + 0 + 0 \\ &= \beta_p^2 \sigma_m^2 + \sigma_{ep}^2 \end{aligned} \quad (8)$$

2.7 Value at Risk

Value at Risk is a statistical concept used as a measurement method to estimate the worst possible loss under normal market conditions over a period of time with a certain level of confidence. The conventional way to measure VaR is to assume that portfolio returns follow a normal distribution.

Value at Risk (VaR) on a portfolio consisting of various assets can be calculated with the equation:

$$VaR = \alpha \sigma_p W \sqrt{t} \quad (9)$$

2.8 *Exponentially Weighted Moving Average*

The Exponentially Weighted Moving Average (EWMA) developed by JP Morgan is an approach used to overcome the inconstant volatility of data by giving greater weight to the most recent data compared to the previous period (Buchdadi, 2008). The EWMA approach assumes that current projections are influenced by projections and actuals in the previous period. The EWMA equation can be written as follows:

$$\sigma_t = \lambda \sigma_{t-1} + (1 - \lambda) R_{t-1} \quad (10)$$

3. Methodology

The type of research used in this research is applied research with research data is secondary data consisting of monthly closing price data listed in the Jakarta Islamic Index (JII), starting October 2018 - October 2023 obtained from the Yahoo Finance website, the composite stock price index (JCI) for the period October 2018 - October 2023 obtained from the Indonesian stock exchange report through the Yahoo Finance website and the interest rate (SBI) for the period October 2018 - October 2023 accessed through www.bi.go.id.

The stages of data analysis in this study are presented with a flowchart in the following figure:

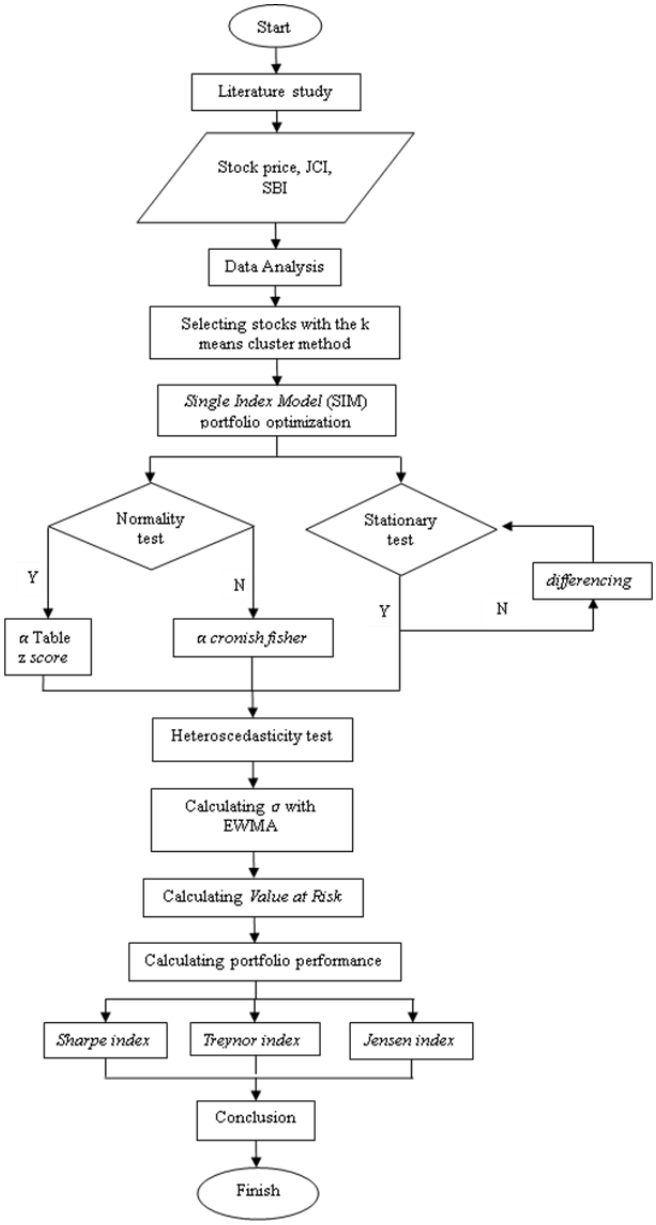

Figure 1 – Research Flowchart

4. Result and Discussion

4.1 Stock Selection using K-Means Cluster

The stock selection process is done by clustering based on the standard deviation of stock returns and the average volume of stock sales. Determination of the number of clusters using the WSS (Within Sum of Square) method obtained the optimal number of clusters based on the following chart:

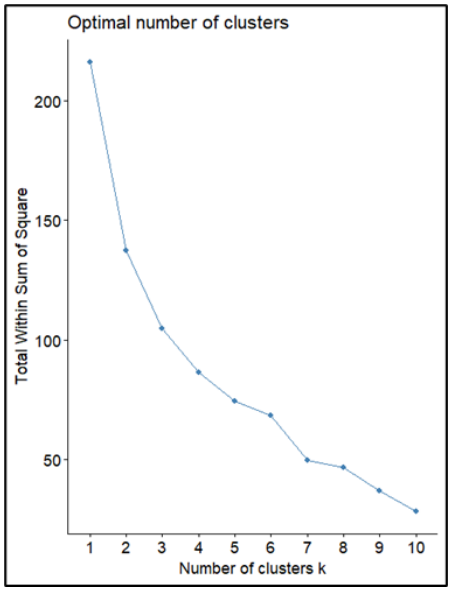

Figure 2 – Optimal Cluster

Based on Figure 2, the curve starts to slope and forms an angled triangle at k=3, so 3 clusters are selected in the stock grouping. The results of the stock clustering are based on the following figure:

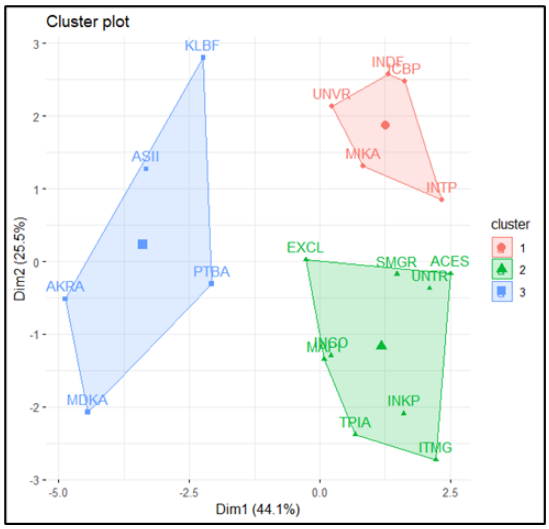

Figure 3 – Stock Clustering Results

Based on Figure 3, clustering stocks using K-means produces 3 optimal cluster groups with each cluster member based on the following table:

Table 1. Cluster Group Members

Cluster	Number of Members	Stock Group
1	**5**	ICBP, INDF, INTP, MIKA, UNVR
2	**9**	ACES, EXCL, INCO, ITMG, INKP, MAPI, TPIA, SMGR, UNTR
3	**5**	ASII, AKRA, KLBF, MDKA, PTBA

4.2 Optimal Portfolio using Single Index Model

1. Calculating realized return (R_i) and expected return $(E(R_i))$

The positive expected return value is used as a reference to determine which stocks are included in the further analysis stage to determine the optimal stock portfolio. The results of the calculation of realized return (R_i) and expected return $(E(R_i))$ selected stocks in the selection process using K-Means Clustering are as follows:

Table 2. Realized Return and Expected Return of JII Stock Index

Cluster	No	Stock Code	R_i	$E(R_i)$
	1	ICBP	0.27321	0.00455
	2	INDF	0.23313	0.00389
1	3	INTP	-0.39654	-0.00661
	4	MIKA	0.867682	0.01446
	5	UNVR	-0.72618	-0.01210
	6	ACES	0.56983	0.00950
	7	EXCL	0.19971	0.00333
	8	INCO	0.98918	0.01649
	9	ITMG	0.71323	0.01189
2	10	INKP	0.27970	0.00466
	11	MAPI	1.28483	0.02141
	12	TPIA	1.52807	0.02547
	13	SMGR	-0.01494	-0.00025
	14	UNTR	0.02787	0.00046
	15	ASII	-0.07251	-0.00121
	16	AKRA	1.14910	0.01915
3	17	KLBF	0.32136	0.00536
	18	MDKA	1.93494	0.03225
	19	PTBA	-0.21798	-0.00363

2. Calculating the market return (R_m) and expected return market $(E(R_m))$

Table 3. Market Return and Expected Return Market

IHSG	
R_m	$E(R_m)$
0.19642	0.00327

Based on Table 3, the market expected return was able to record a positive value of 0.00327 in the period October 2018 - October 2023.

3. Calculated the variance of stock return (σ_i^2) dan variance return market (σ_m^2)

In investment management, it is closely related to risk, which is the uncertainty of the level of return that will be obtained by investors. Risk describes the possibility of deviation of realized return and expected return. The results of the calculation of individual stock risk and market risk are presented in the following table:

Table 4. Variance of Stock Return and Variance Return Market

Cluster	Stock Code	σ_i^2		σ_m^2
	ICBP	0.00417		
	INDF	0.00422	**JCI**	0.00160
1	MIKA	0.00750		
	ACES	0.01239		
	EXCL	0.01136		
	INCO	0.01474		
	ITMG	0.02271		
	INKP	0.01911		
2	MAPI	0.01655		
	TPIA	0.02290		
	UNTR	0.01072		
	AKRA	0.01251		
3	KLBF	0.00377		
	MDKA	0.01694		

4. Calculating Covariance R_i and R_m (σ_{im})

Stock covariance and market covariance show the relationship between stock returns and market returns. The results of the calculation of stock covariance and market covariance are presented in the following table:

Table 5. Stock Covariance and Market Covariance

Cluster	Stock Code	σ_{im}
	ICBP	1.36191E-05
1	INDF	0.00046856
	MIKA	0.00046263
	ACES	0.001854315
	EXCL	0.001788467
	INCO	0.003104995
	ITMG	0.002942658
2	INKP	0.002748029
	MAPI	0.002874841
	TPIA	0.002122533
	UNTR	0.00145088
	AKRA	0.003072111
3	KLBF	0.000844758
	MDKA	0.002524204

5. Calculating beta (β_i) and alpha (α_i) of each stock

The results of the calculation of beta (β_i) and alpha (α_i) of each stock are presented in the following table:

Table 6. *Beta* (β_i) and *Alpha* (α_i) Stocks

Cluster	Stock Code	β_i	α_i
	ICBP	0.00851	0.004525638
1	INDF	0.29286	0.002926705
	MIKA	0.28915	0.013514792
	ACES	1.15897	0.005703119
	EXCL	1.11781	-0.000330879
	INCO	1.94065	0.010133401
	ITMG	1.83919	0.005866378
2	INKP	1.71755	-0.000960962
	MAPI	1.79681	0.015531702
	TPIA	1.32661	0.021124972
	UNTR	0.90682	-0.002504174
	AKRA	1.92010	0.012865878
3	KLBF	0.52798	0.003627642
	MDKA	1.57765	0.027084306

Based on Table 6, it is known that the stock with the highest beta is INCO of 1.94065, which shows that INCO shares move very actively against changes in market indices. Meanwhile, the lowest beta value is obtained by ICBP shares of 0.00851 which indicates that ICBP shares tend to be less sensitive to changes in market indices.

6. Calculating the variance residual error/unsystematic (σ_{ei}^2)

Stock residual error variance is an unsystematic risk that can be eliminated by diversification. The results of the calculation of variance residual error are presented in the following table:

Table 7. Variance Residual Error/Unsystematic (σ_{ei}^2)

Cluster	Stock Code	σ_{ei}^2
1	ICBP	0.00417
	INDF	0.00409
	MIKA	0.00737
2	ACES	0.01024
	EXCL	0.00936
	INCO	0.00872
	ITMG	0.01730
	INKP	0.01439
	MAPI	0.01139
	TPIA	0.02009
	UNTR	0.00941
3	AKRA	0.00661
	KLBF	0.00333
	MDKA	0.01296

7. Calculating the risk-free rate return (R_f)

The calculation of risk-free returns is obtained from the average monthly SBI interest rate during the study period, namely October 2018 - October 2023. The results of the risk-free return calculation are presented in the following table:

Table 8. Risk-Free Rate Return (R_f)

	Risk Free Rate	
Date	**BI-7Day-RR**	**Month**
Oktober 2023	6.00%	0.005
September 2023	5.75%	0.004792
Agustus 2023	5.75%	0.004792
⋮	⋮	⋮
Desember 2018	6.00%	0.005
November 2018	6.00%	0.005
Oktober 2018	5.75%	0.004792
R_f		0.003883

Based on the calculation results, Table 8 shows that the risk-free return for the last 5 years starting from the October 2018-October 2023 period is 0.003883.

8. Calculating Excess Return to Beta (ERB)

The ERB value is a number that is used as a basis for determining whether a stock can be included in the optimal portfolio. The results of the ERB calculation are presented in the following table:

Table 9. Excess Return to Beta (ERB)

Cluster	Stock Code	ERB
1	ICBP	0.07875
	INDF	7.58776E-06
	MIKA	0.03658
2	ACES	0.00484
	EXCL	-0.00050
	INCO	0.00649
	ITMG	0.00435
	INKP	0.00045
	MAPI	0.00976
	TPIA	0.01627
	UNTR	-0.00377
3	AKRA	0.00795
	KLBF	0.00279
	MDKA	0.01798

The results of the ERB calculation show that there are 2 stocks that have negative ERB values, namely EXCL and UNTR so that these stocks are not included in further calculations, because EXCL and UNTR have an expected return value that is smaller than the risk-free return.

9. Stock selection based on Excess Return to Beta (ERB)

Determination of the cut-off point is obtained by determining the values A_i, B_i, and C_i. The results of the calculation of the cut-off point value of stocks included as optimal portfolio candidates are as follows:

Table 10. Excess Return to Beta (ERB)

Cluster	Stock Code	$\mathbf{A_i}$	$\mathbf{B_i}$	C_i	ERB
1	ICBP	0.00137	0.01738	0.0000	0.07875
	INDF	0.00016	20.9831	0.0000	0.0000
	MIKA	0.41522	11.3498	0.0007	0.03658
2	ACES	0.63518	131.129	0.0008	0.00484
	INCO	2.80626	432.11	0.0027	0.00649
	ITMG	0.85115	195.58	0.00104	0.00435
	INKP	0.09292	205.014	0.00011	0.00045
	MAPI	2.76585	283.487	0.00304	0.00976
	TPIA	1.42551	87.613	0.00200	0.01627
3	AKRA	4.4358	557.831	0.00375	0.00795
	KLBF	0.23372	83.7836	0.00033	0.00279
	MDKA	3.45324	192.063	0.00423	0.01798

Based on Table 10, it can be seen that the cut-off point value of 0.00423 which is the highest C_i value, is used as a limit point in determining the stocks included in the optimal portfolio. Of the 12 selected stocks, 9 stocks are included in the formation of the optimal portfolio, namely ICBP, MIKA, ACES, INCO, ITMG, MAPI, TPIA, AKRA and KLBF.

10. The proportion of founds (W_i) and scale-weighted (Z_i) of each stock

The results of the calculation of the proportion of funds and weighted scale on each stock are presented in the following table:

Table 11. Fund Proportion (W_i) and Scale Weighted (Z_i) of Each Stock

Cluster	Stock Code	Z_i	W_i
1	ICBP	0.152181091	0.023648
	MIKA	1.270110916	0.197371
2	ACES	0.069875331	0.010858
	INCO	0.504987692	0.078474
	ITMG	0.013353921	0.002075
	MAPI	0.872511467	0.135586
	TPIA	0.795435058	0.123608
3	AKRA	1.674332175	0.168193
	MDKA	1.082344415	0.260186

Based on Table 11, it can be seen that the highest proportion of funds is MDKA stock of 0.26018614 or around 26%, while, the stock that has the lowest proportion of funds is ITMG of 0.002075159 or around 0.2% of all stocks that are candidates for optimal portfolios. Stocks that have the highest proportion of funds can be an alternative investment that can be chosen by investors because seen from the proportion of funds will get a large profit with adjusted risk.

11. Calculating of portfolio return and risk of the optimal stock portfolio

The portfolios formed from the application of the Single Index Model (SIM) method is 5 portfolios, consisting of the optimal portfolio of all candidate stocks, the optimal portfolio of representative stocks of each cluster, the optimal portfolio of cluster 1 stock, the optimal portfolio of cluster 2 stocks and optimal portfolio of cluster 3 stocks. The results of the calculation of the return and risk of each portfolio are presented below:

Table 12. Portfolio Return and Risk

	$E(R_p)$	Risk (σ_p^2)
Portfolio 1	0.022047	0.01277
Portfolio 2	0.02504	0.01471
Portfolio 3	0.01343	0.00714
Portfolio 4	0.01951	0.01761
Portfolio 5	0.02612	0.01482

Based on Table 12, it can be seen that the expected return portfolio ($E(R_p)$) of portfolio 1 is 0.022047 or 2.20%. This means that investors who invest their funds in portfolio 1 will benefit by 2.20% per day from the amount of funds that have been invested. Looking at the value of the expected return of the portfolio. considered quite promising because it is above the market return level of 0.00327 and the average risk-free return of 0.00388. With the amount of portfolio risk formed is 0.01277, meaning that investors will get a risk of 1.2% when investing their funds in portfolio 1.

4.3 *Value at Risk* (VaR) Calculating using *Exponentially Weighted Moving Average* (EWMA)

1. Statistical test of stock return

Before calculating VaR, the return data of each stock is tested to determine the characteristics of the return of each stock. Tests carried out include stationary test, normality test and heteroscedasticity test.

a. Stationary test

The stationary data return test uses the Augmented Dicky Fuller Test (ADF) with the help of R-Studio software. The ADF test criteria with a critical value of 0.05 and a 95% confidence interval are as follows:

$H_0: \delta = 0$ (stock return are not stationary)
$H_1: \delta \neq 0$ (stationary stock return)

If $P \leq 5\%$ or ADF < CV 5%. then H_0 is rejected meaning the return data is stationary, while if P≥5% or ADF > CV 5%, then H_0 is accepted meaning the return data is not stationary. The Augmented Dicky Fuller Test (ADF) test results on stock returns are presented in the following table:

Table 13. Stationary Test Result of Stock Return Data

Cluster	Stock Code	*ADF-Test*	*P-Value*	Conclusion
1	ICBP	-4.4771	0.01	Stationary
	MIKA	-4.4652	0.01	Stationary
	ACES	-3.5566	0.04447	Stationary
	INCO	-4.0654	0.01283	Stationary
2	ITMG	-3.2856	0.08245	Not Stationary
	MAPI	-6.4463	0.01	Stationary
	TPIA	-3.8275	0.02324	Stationary
3	AKRA	-5.1501	0.01	Stationary
	MDKA	-4.6914	0.01	Stationary
	Portfolio 1	-4.337	0.01	Stationary
	Portfolio 2	-4.4582	0.01	Stationary
	Portfolio 3	-4.3527	0.01	Stationary
	Portfolio 4	-4.4318	0.01	Stationary
	Portfolio 5	-4.3028	0.01	Stationary

Based on Table 13, it can be seen that there is 1 stock that is not stationary, namely ITMG shares, so it is necessary to do the differencing process. The results of the ITMG stock stationary test after differencing are presented in the following table:

Table 14. Stationary Test Result of Stock Return Data with Differencing

	ITMG
ADF Test	-6.9791
P-Value	0.01
Conclusion	Stationary

Based on the results of differencing by ITMG shares, the p-value is smaller than 0.05, meaning that the ITMG stock return data is stationary.

b. Normality test

The test statistic used is the Jarque Bera value which is compared to the chi-square (χ^2) ($\alpha = 5\%. df = 2$) of 5.991. The test hypothesis used is as follows:
H_0: return data is normally distributed
H_1: return data is not normally distributed

H_0 is rejected if the JB > χ^2 value with *df* = 2 and *p-value* < α. The Jarque Bera normality test results for the return data are presented in the following table:

Table 15. Jarque-Bera Normality Test Results Stock Return Data

Cluster	Stock Code	*JB*	*Chi-Square*	*P-value*	Conclusion
	ICBP	0.13834	5.991	0.9332	Normal
1	MIKA	10.387	5.991	0.0055	Not Normal
	ACES	0.45485	5.991	0.7966	Normal
	INCO	1.8793	5.991	0.3908	Normal
2	ITMG	28.797	5.991	5.582e-07	Not Normal
	MAPI	5.1404	5.991	0.0765	Normal

Cluster	Stock Code	*JB*	*Chi-Square*	*P-value*	Conclusion
3	TPIA	107.19	5.991	2.2e-16	Not Normal
	AKRA	1.181	5.991	0.5541	Normal
	MDKA	0.589	5.991	0.7449	Normal
	Portfolio 1	9.3017	5.991	0.0096	Not Normal
	Portfolio 2	1.7015	5.991	0.4271	Normal
	Portfolio 3	10.017	5.991	0.0067	Not Normal
	Portfolio 4	37.933	5.991	5.794e-09	Not Normal
	Portfolio 5	1.6429	5.991	0.4398	Normal

Based on Table 15, it can be seen that there are 6 return data that are not normally distributed, including MIKA, ITMG, TPIA, Portfolio1, Portfolio 3 and Portfolio 4, because it has a Jarque Bera value > Chi-Square and P-value < α (0.05) so that the α value on the return data must be corrected first using the Cornish-Fisher Expansion (α') approach with a value of α (5%)=1.645. A data is normal if the value of α'> coefficient of skewness. The results of the calculation α' are presented in the following table:

Table 16. Normality Test Results of Stock Return Data

Cluster	Stock Code	Koef. *Skewness*	Z-score	Z-correction α'	Conclusion
1	MIKA	0.56681	1.645	1.48383	Normal
2	ITMG	0.24550	1.645	1.57520	Normal
	TPIA	0.42013	1.645	1.52554	Normal
	Portofolio 1	-0.08257	1.645	1.66848	Normal
	Portofolio 3	0.54047	1.645	1.49132	Normal
	Portofolio 4	-0.02848	1.645	1.65310	Normal

Based on Table 16, The value of α' for return data after correction shows $\alpha' >$ kewness coefficient so that all return data is normally distributed.

c. Heteroscedasticity test

The heteroscedasticity test uses White's heteroscedasticity test. If the variance of return data is constant (homoscedasticity) then the volatility value is calculated using standard deviation. but if the variance of return data is not constant (heteroscedasticity) then volatility is calculated using the Exponentially Weighted Moving Average (EWMA) approach. The test hypothesis used for White's Heteroscedasticity test is:

H_0: σ return is homoscedasticity

H_1: σ return is heteroscedasticity

If P-value ≤ 5% reject H_0 meaning σ return is heteroscedasticity and if P-value > 5% accept H_0 meaning σ return is homoscedasticity. The results of heteroscedasticity testing using White's Heteroscedasticity test are presented in the following table:

Table 17. White's Heteroscedasticity Test Results

Cluster	Stock Code	P-value	Conclusion
1	ICBP	0.06681	Homoskedastisitas
	MIKA	0.01077	Heteroskedastisitas
	ACES	0.5243	Homoskedastisitas
	INCO	0.02385	Heteroskedastisitas
2	ITMG	0.005474	Heteroskedastisitas
	MAPI	0.1619	Homoskedastisitas
	TPIA	0.00326	Heteroskedastisitas
3	AKRA	0.2428	Homoskedastisitas
	MDKA	0.2383	Homoskedastisitas
	Portofolio 1	0.1876	Homoskedastisitas
	Portofolio 2	0.8879	Homoskedastisitas
	Portofolio 3	0.01062	Heteroskedastisitas
	Portofolio 4	0.645	Homoskedastisitas
	Portofolio 5	0.2215	Homoskedastisitas

Based on Table 17, volatility measurement with the Exponentially Weighted Moving Average (EWMA) approach is carried out on 4 stocks and 1 optimal portfolio because it has a variant of return that is not constant (Heteroscedasticity).

2. Optimal portfolio with exponentially weighted moving average approach

VaR calculation uses a time horizon with a significance level of 95% (α (0.05) which is 1.645) assuming that the investment value is IDR 100,000,000. A summary of the results of the calculation of the expected return of the portfolio, the risk of the portfolio formed, the performance of the portfolio and the amount of potential loss calculated by the Value at Risk (VaR) method are presented in the following table:

Table 18. Optimal Portfolio with Exponentially Weighted Moving Average Approach

	$E(R_p)$	Risk (σ_p^2)	Portfolio Performance	Exposur	VaR (1 Day)	VaR (1 Month)
Portfolio 1	0.022047	0.01277	0.160720		3,083,287.56	6,677,589.25
Portfolio 2	0.02504	0.01471	0.17445		3,553,167.10	7,695,224.65
Portfolio 3	0.01343	0.00714	0.11301	100jt	2,633,286.63	5,703,005.68
Portfolio 4	0.01951	0.01761	0.11772		3,757,870.17	8,138,557.60
Portfolio 5	0.02612	0.01482	0.18266		4,005,564.17	8,674,997.61

Based on Table 18, the selection of the right portfolio depends on the investor's risk tolerance. Investors with low risk tolerance prefer portfolios with lower returns and smaller risks. so they tend to choose Portfolio 3. Investors invest their funds in Portfolio 3 because it has a positive expected return of 0.01343 which is the lowest among other portfolios and the

lowest risk of 0.00714 among all portfolios, but still need to consider VaR to ensure investors receive the portfolio with a maximum potential loss at the 95% confidence interval of IDR 2,633,286.63 in the 1-day investment time interval and IDR 5,703,005.68 in the 1-month investment time interval.

Investors with moderate risk tolerance can accept investment values that go up and down to get higher returns, so Portfolio 1 can be the choice of investors in investing their funds. Portfolio 1 offers a good balance between return and risk., with an expected return of 0.022047 which is higher than Portfolio 3 and a slightly higher risk of 0.01277. However, it is still necessary to consider VaR to ensure investors accept the portfolio with a maximum potential loss at 95% confidence interval of IDR 3,083,287.56 in 1 day investment time interval and IDR 6,677,589.25 in 1 month investment time interval.

Investors with high risk tolerance are willing to accept greater risk to obtain higher potential returns, so that Portfolio 2 can be the choice of investors in investing their funds. Portfolio 2 offers the highest expected return among all portfolios of 0.02504 but also has a high risk of 0.01471, as well as the largest potential loss of IDR 3,553,167.10 at an investment time interval of 1 day and IDR 7,695,224.65 at an investment time interval of 1 month. Investors who invest their funds in Portfolio 2 tend to be more courageous and ready to accept the consequences of the possibility of greater losses seen based on VaR.

5. Conclusion

The analysis results show that there are 5 portfolios formed. The best portfolio that can be chosen by investors depends on the investor's risk tolerance. Investors with low risk tolerance can choose Portfolio 3 with expected return 0.01343 and risk 0.00714 and VaR Rp2,633,286.63. Investors with moderate risk tolerance can choose Portfolio 1 with expected return 0.022047. risk 0.01277 and VaR IDR3,083,287.87. Investors with high risk tolerance can choose Portfolio 2 with an expected return of 0.02504 and a risk of 0.01471 and VaR of IDR3,553,167.10.

The results of the calculation of the performance analysis of the 5 portfolios formed using the Sharpe, Treynor and Jensen methods, the performance results of the three methods used have a positive performance value on all portfolios formed, this indicates that the portfolio formed shows good portfolio performance. The highest performance of the three methods used is the Sharpe indeks, so that the method is the best performance method when compared to the Treynor index and Jensen index because the Sharpe index is able to minimize all risks that exist systematically or unsystematically in a combination of stock portfolios.

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