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Crisping the Fuzzy ARIMA Intervals of Possibility for Short Term Forecasts

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A B S T R A C T

The Fuzzy autoregressive integrated moving average (FARIMA) model is a fuzzy-enhanced version of the autoregressive integrated moving average (ARIMA) model that yield improved prediction accuracy with fewer data observations as compared to the classical ARIMA models. The FARIMA time series utilizes membership functions of the fuzzy coefficients and generates forecasts in the form of possibility intervals. However, the FARIMA model does not provide crisp forecast values for forecasting future possibility intervals. This paper aims to simultaneously achieve in-sample and out-sample intervals of possibility forecasts by converting Fuzzy ARIMA possibility intervals into crisp values. The method is tested on exchange rate of the New Taiwan Dollar (NTD) against the United States Dollar (USD) and the annual average mean surface air temperature of Nigeria. The results demonstrate that the proposed method produces out-of-sample possibility interval forecasts that closely align with those obtained using observed values in most cases. In addition, forecasts performance evaluation results indicate that the proposed method produces smaller MAPE and RMSE values in LB predictions while approximately competing in UB predictions compared to the considered methods in the literature. Moreover, the proposed method has advantage of forecasting future possibility intervals without relying on crisp out-of-sample observed values. This implies the method could aid policy makers in determining the worst and best projected bounds that could be used for making future decisions without actual out-of-sample crisp observations.

Keywords: ARIMA, Crisp Forecast Values, FARIMA, Fuzzy Time Series, Intervals of Possibility

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1. Introduction

Classical time series forecasting approaches provide insights into future trends based on historical records. This characteristic makes these techniques widely applicable in various domains, such as finance, economics, and engineering. The Autoregressive Integrated Moving Average (ARIMA) approach proposed by [1], is one of the most result oriented, well-known and widely used classic time series forecasting approach in the literature. The approach is considered a milestone in time series forecasting [2]. Additionally, it is frequently used as a fundamental method in time series forecasting to develop parsimonious ARIMA models [3], and has proven effective for various forecasting problems [4]. In addition, the approach has demonstrated accurate results in short time forecasting problems [5], [6].

The ARIMA model assumes that both current and past observations, along with past innovations define linear functional relationship of future observations. This assumption could limit the model in some real life applications. Linearity and large history data requirement are the two basic limitations of the ARIMA model [7]. According to the data limitation, ARIMA model requires at least fifty, and preferably above one-hundred observations [4], [8]. Furthermore, the ARIMA models cannot deal with forecasting problems in which the time series data are linguistic observations [9].

In recent studies, a fuzzy-enhanced version of the autoregressive integrated moving average approach was considered in the literature. This hybrid approach is known as fuzzy autoregressive integrated moving average (FARIMA) model. This model is applicable for prediction under conditions of large data availability constraints. Hence, the FARIMA model requires little obtainable data than a classic ARIMA model [10]. Moreover, unlike ARIMA, the FARIMA model does not require assumptions about error terms due to the fuzziness of its outputs.

The introduction of the FARIMA model has expanded the incorporation of fuzzy sets theory presented in [11] into forecasting problems. Numerous studies have suitably applied fuzzy sets in solving forecasting problem in the literature including but not limited to [12], [13], [14]. The Fuzzy time series (FTS) concepts proposed in [15] are based on fuzzy set theory. It was one of earliest study that integrated fuzzy sets into time series forecasting approaches in the literature [16], [17], [18], [19].

Leveraging on the works of time-series ARIMA (p, d, q) models and fuzzy regression methods presented by [20], the foundation for the FARIMA model was formulated in [21]. The authors tested the appropriateness and effectiveness of the model on foreign exchange rate of the New Taiwan (NT) dollar against the United States dollar, and the results yields good forecasts. Subsequently, several studies have utilized the fuzzy ARIMA model to address time series forecasting problems: Reyes et al. [22] proposed a new hybrid fuzzy time series model based on Fuzzy ARIMA method that achieved better in-sample and out-sample accuracy tests. The paper concluded that fuzzy time series models can estimate the behavior of volatile variables such as exchange rates. An application of the FARIMA approach on gold prices forecasting indicates that the FARIMA model outperforms the ARIMA model, as evinced by minimum mean square error (MSE) and root mean square error (RMSE) values [23].

FARIMA model on traffic in cloud computing findings indicate that the model is capable of producing predictions with high precision [10]. Using the basic tenets of the fuzzy autoregressive integrated moving average models, the results in [4] forecasted the Iran's steel consumption with improved precision . Similarly, in [24], the authors proposed a FARIMA model that can be used as an alternative model to steel consumption forecasting on the account of Iran's steel consumption data. The application of FARIMA method on simulated time series data is illustrated in [5]. Moreover, forecasting of the worst and best return in Stock market is demonstrated through FARIMA [25]. According to the authors, investors and economic policy makers can forecast the minimum and maximum return in Stock market using the FARIMA model.

According to [10], it is worthy to note that due to uncertainty of the environment, future trends must be predicted within a short period of time and based on less data. Thus, the applications of models with large history data requirement is inappropriate amidst global contemporary technological advancements in which changes in the current and past values over time is rapid and dramatic. However, the use of fuzzy parameters in the FARIMA model reduces the requirements of large historical data to forecast future values. Furthermore, empirical results indicated the FARIMA method have outperformed the ARIMA method which is known to require large history data (at least fifty, and preferably above one-hundred observations [4], [8]).

The authors in [26] learned that FARIMA method has significantly narrowed prediction intervals relative to the ARIMA and Fuzzy Linear the Regression methods. Abdelkader et al. [23] determined FARIMA model as the best in forecasting gold prices on the basis of predictive precision measures represented by MSE and RMSE as the values of these measures are less than what is in the ARIMA model. Similarly, Torbat et al. [24] reported that the width of the forecasted interval in the FARIMA model indicates 32.95% improvement upon the possible interval of the ARIMA (95% Confidence Intervals) in Iran's steel consumption forecasting case. Moreover, the width of the forecasted interval by the FARIMA model is narrower than ARIMA (95% Confidence Interval) obtained in the US dollar, Euro, and British pound exchange rate forecasting cases [27]. Furthermore, mean square error value reported in [28] indicates that the forecasting interval value from Fuzzy ARIMA model is better than ARIMA model.

The current Fuzzy autoregressive integrated moving average methods utilizes fuzzy coefficients and generate forecasts in form of intervals of possibility. Consequently, the FARIMA method does not provide a crisp forecast values to make future decisions [29], [22]. Hence, there is room to explore various possibilities of enhancement of the method in order to simultaneously achieve in-sample and future intervals of possibility forecasts that could be used for making future decisions.

The remainder of this paper is structured as follows: In section 2 of the study, foundation concepts and the proposed crisping method are explained. In section 3, empirical Results and Discussion with the aid of tables and figures are presented. Finally, section 4 provides a conclusion that summarizes the findings.

2. Model Formulation

2.1. Fuzzy Autoregressive Integrated Moving Average (FARIMA) Model

The FARIMA model follows a structure similar to fuzzy regression, with lagged values of the response variable and associated residuals as explanatory variables. The parameters of ARIMA (p, d, q) model form the structural basis of the FARIMA (p, d, q) model. Thus, p is the order of the Autoregressive terms, q is the order of the Moving Average terms, while d is the differencing order needed to achieve stationarity of the time series.

The following is the generalized FARIMA (p, d, q) model built on the ARIMA process of a time series Y_t, $t = 1, 2, \ldots, k$:

$$\begin{aligned} \widetilde{\omega}_p(L) Y_t^* &= \widetilde{\tau}_q(L)\varepsilon_t, && (1)\\ Y_t^* &= \Delta^d (Y_t - \mu). && (2) \end{aligned}$$

The expanded form of equation (1) is given in equation (3):

$$\widetilde{Y}_t^* = K + \widetilde{\omega}_1 Y_{t-1}^* + \widetilde{\omega}_2 Y_{t-2}^* + \cdots + \widetilde{\omega}_p Y_{t-p}^* + \varepsilon_t - \widetilde{\tau}_1 \varepsilon_{t-1} - \widetilde{\tau}_2 \varepsilon_{t-2} - \cdots - \widetilde{\tau}_q \varepsilon_{t-q} \tag{3}$$

where, equation (2) is the ARIMA process of the time series Y_t, $\Delta = 1 - L$, the difference operator, L is a lag operator; generally, $L^n Y_t = Y_{t-n}$, Y_t are observations, K is a constant determined as $\hat{\mu}\left(1 - \sum_{i=1}^{p} \hat{\omega}_i\right)$, $\hat{\mu}$ is mean of a stationary time series Y_t^*, while $\widetilde{\omega}_1, \widetilde{\omega}_2, \ldots, \widetilde{\omega}_q$ and $\widetilde{\tau}_1, \widetilde{\tau}_2, \ldots, \widetilde{\tau}_q$ are fuzzy numbers. In addition, autocorrelation function (ACF) and partial autocorrelation function (PACF) are the key tools used to develop the structure of the ARIMA model. Thus, in a FARIMA (p, d, q) model, the PACF determines p of the autoregressive (AR) component and the ACF determines q of the

Moving Average component of the ARIMA model. The plots of these functions are compared to their corresponding theoretical behaviors when the differencing order is known.

Equation (3) can also be rewritten in terms of generalized fuzzy numbers as follows:

$$\widetilde{Y}_t^* = K + \widetilde{A}_1 Y_{t-1}^* + \widetilde{A}_2 Y_{t-2}^* + \cdots + \widetilde{A}_p Y_{t-p}^* + \varepsilon_t - \widetilde{A}_{p+1}\varepsilon_{t-1} - \widetilde{A}_{p+2}\varepsilon_{t-2} - \cdots - \widetilde{A}_{p+q}\varepsilon_{t-q} \tag{4}$$

where, $\widetilde{A}_1, \widetilde{A}_2, \ldots \widetilde{A}_p, \widetilde{A}_{p+1}, \widetilde{A}_{p+2}, \ldots \widetilde{A}_{p+q}$ are fuzzy numbers. For each type of $\widetilde{A}_j$, the membership functions are assumed to be triangular. By definition, it can be expressed as:

$$\mu_{\widetilde{A}_j}(A_j) = \begin{cases} 1 - \dfrac{|c_j - A_j|}{w_j} & \text{if } c_j - w_j \le A_j \le c_j + w_j \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where c_j is a centre, w_j is a width of the fuzzy number. Based on the extension principle [30], the membership function of the fuzzy Y_t^* is given in equation (6):

$$\mu_{\widetilde{Y}^*}(Y_t^*) = \begin{cases} 1 - \dfrac{\left| Y_t^* - \sum\limits_{j=0}^{p} c_j Y_{t-j}^* - \varepsilon_t + \sum\limits_{j=p+1}^{p+q} c_j \varepsilon_{t+p-j} \right|}{\sum\limits_{j=0}^{p} w_j |Y_{t-j}^*| + \sum\limits_{j=p+1}^{p+q} w_j |\varepsilon_{t+p-j}|} & \text{for } Y_t^* \neq 0,\ \varepsilon_t \neq 0 \\ 0 & \text{otherwise} \end{cases}. \tag{6}$$

The membership value of each observation Y_t is considering the condition that should be greater than an imposed threshold h, $h \in [0, 1]$ [26]. The $h-$level denotes membership function threshold level degree that should be satisfied by all observations $y_1, y_2, \ldots, y_k$. This implies:

$$\mu_Y(Y_t) \ge h \tag{7}$$

The choice of the h value will influence the widths, w_j, of the fuzzy parameters.

A FARIMA (p, d, q) problem seeks to find the coefficients $\tilde{A}_j = [c_j, w_j]$ that minimize the fuzziness S included in the model for all data sets. Mathematically, this becomes:

$$Min\ S = \sum_{t=1}^{k}\sum_{j=1}^{p} w_j |\varphi_{jj}||Y_{t-j}^*| + \sum_{t=1}^{k}\sum_{j=p+1}^{p+q} w_j |\rho_{j-p}||\varepsilon_{t+p-j}| \tag{8}$$

where: w_j, the spread around the center of the fuzzy number; φ_{jj}, the partial autocorrelation coefficient of time lag j; ρ_{j-p}, the autocorrelation coefficient of time lag $j-p$; $Y_t^* = \Delta^d(Y_t - \mu)$; and ε_t, ARIMA residuals at time t. The residuals, ε_t, are assumed to be independently and identically distributed with a mean of zero and a constant variance (σ^2).

The parameters of a FARIMA (p, d, q) can be obtained (for derivation, see [31]) in what follows:

$$\begin{aligned} &\min S = \sum_{t=1}^{k}\sum_{j=1}^{p} w_j |\varphi_{jj}|\ |Y_{t-j}^*| + \sum_{j=p+1}^{p+q} w_j |\rho_{j-p}|\ |\varepsilon_{t+p-j}| \\ &\text{s.t.}\quad \sum_{j=1}^{p} c_j Y_{t-j}^* + \varepsilon_t - \sum_{j=p+1}^{p+q} c_j \varepsilon_{t+p-j} + (1-h)\sum_{j=1}^{p} w_j \left|Y_{t-j}^*\right| + \sum_{j=p+1}^{p+q} w_j \left|\varepsilon_{t+p-j}\right| \ge Y_t^*, \quad \forall t = 1, \ldots, k \\ &\qquad \sum_{j=1}^{p} c_j Y_{t-j}^* + \varepsilon_t - \sum_{j=p+1}^{p+q} c_j \varepsilon_{t+p-j} - (1-h)\sum_{j=1}^{p} w_j \left|Y_{t-j}^*\right| + \sum_{j=p+1}^{p+q} w_j \left|\varepsilon_{t+p-j}\right| \le Y_t^*, \quad \forall t = 1, \ldots, k \\ &\qquad w_j \ge 0, \quad \forall j = 1, \ldots, p+q. \end{aligned} \tag{9}$$

It is important to note that the procedure of finding the FARIMA (p, d, q) parameters was formulated as a Linear Programming problem (LP). After the solution of the LP problem, fuzzy intervals model is presented as follows:
Lower Bond (LB):

$$Y_{(LB)} = K + \sum_{j=1}^{p}(c_j - w_j)Y^*_{t-j} + \varepsilon_t - \sum_{j=p+1}^{p+q}(c_j - w_j)\varepsilon_{t+p-j} \tag{10}$$

Upper Bond (UB):

$$Y_{(UB)} = K + \sum_{j=1}^{p}(c_j + w_j)Y^*_{t-j} + \varepsilon_t - \sum_{j=p+1}^{p+q}(c_j + w_j)\varepsilon_{t+p-j}. \tag{11}$$

2.2. Proposed Model Formulation

The FARIMA model utilizes triangular fuzzy parameters as coefficients of the predictors and generate in-sample forecasts in form of intervals of possibility. Consequently, crisp forecast values for forecasting out-sample intervals of possibility that could be used for future decisions are not provided. Thus, further enhancements can be explored to improve both in-sample and out-sample interval forecasting for better decision-making.

The crisp forecast values can be generated as follows: Fitting ARIMA and FARIMA models on an observed stationary time series Y^*_t data are the first stages of the proposed model. Next, an out of sample ARIMA forecast values $\hat{Y}^*_{t+i}$ are generated to obtain an out-sample UB prototype forecasts $\hat{\hat{Y}}^*_{t+i}$. Mathematically, the UB prototype forecast model is as follows:

$$\hat{\hat{Y}}^*_{t+i} = K + \sum_{j=1}^{p}(c_j + w_j)\hat{Y}^*_{(t+i)-j} + \varepsilon_{t+i} - \sum_{j=p+1}^{p+q}(c_j + w_j)\varepsilon_{(t+i+p)-j}. \tag{12}$$

The proposed out of sample crisp model is given as:

$$\hat{Y}^{**}_{t+i} = K + \sum_{j=1}^{p}(c_j - w_j)\hat{\hat{Y}}^*_{(t+i)-j} + \varepsilon_{t+i} - \sum_{j=p+1}^{p+q}(c_j - w_j)\varepsilon_{(t+i+p)-j}. \tag{13}$$

where, $i = 1, \ldots m$, m denotes number of out-of-sample forecasts. Hence, the proposed out of sample intervals of possibility prediction equation is in what follows:

$$\begin{aligned}
Y_{t+i(LB)} &= K + (c_1 - w_1)\hat{Y}^{**}_{(t+i)-1} + (c_2 - w_2)\hat{Y}^{**}_{(t+i)-2} + \cdots + (c_p - w_p)\hat{Y}^{**}_{(t+i)-p} + \varepsilon_{t+i} \\
&\quad - (c_{p+1} - w_{p+1})\varepsilon_{(t+i)-1} - (c_{p+2} - w_{p+2})\varepsilon_{(t+i)-2} - \cdots - (c_{p+q} - w_{p+q})\varepsilon_{(t+i)-q} \\
Y_{t+i(UB)} &= K + (c_1 + w_1)\hat{Y}^{**}_{(t+i)-1} + (c_2 + w_2)\hat{Y}^{**}_{(t+i)-2} + \cdots + (c_p + w_p)\hat{Y}^{**}_{(t+i)-p} + \varepsilon_{t+i} \\
&\quad - (c_{p+1} + w_{p+1})\varepsilon_{(t+i)-1} - (c_{p+2} + w_{p+2})\varepsilon_{(t+i)-2} - \cdots - (c_{p+q} + w_{p+q})\varepsilon_{(t+i)-q}
\end{aligned}. \tag{14}$$

For $i \le p$, $\hat{Y}^{**}_{(t+i)-1}, \hat{Y}^{**}_{(t+i)-2}, \ldots, \hat{Y}^{**}_{(t+i)-(p-1)}$ are equivalent to the in-sample crisp forecasts $\left(\hat{\hat{Y}}^*_t\right)$, corresponding to $t, t-1, \ldots, t-(p-1)$ based on UB forecasts obtain as:

$$\hat{\hat{Y}}^*_t = K + \sum_{j=1}^{p}(c_j - w_j)Y_{(UB)t-j} + \varepsilon_{t+i} - \sum_{j=p+1}^{p+q}(c_j - w_j)\varepsilon_{(t+i+p)-j}. \tag{15}$$

3. Result and Discussion

In order to test the appropriateness of the proposed model, we consider the FARIMA model on the application of forecasting the exchange rate of the New Taiwan Dollar (NTD) against the United States Dollar (USD) presented by [21], a foundational study on the FARIMA method as well as the annual average mean surface air temperature of Nigeria [32].

3.1. Application of the Proposed Method to the NTD against the USD

Equations (16) and (17) are the ARIMA and FARIMA models of the data according to [21] respectively with the Z variable in the work substituted by Y.

$$Y_t = 28.0932 + 0.499\, Y_{t-1} - 0.519\, Y_{t-2} + a_t, \tag{16}$$

$$\widetilde{Y}_t = 28.0932 + (0.499,\ 0.0004)\, Y_{t-1} + (0.519,\ 0)\, Y_{t-2} + a_t. \tag{17}$$

Equation (18) defines the upper bound (UB) prototype forecast using out of sample ARIMA inputs, while equation (19) presents the crisp forecast model. The results demonstrate that the proposed method produces crisp out-of-sample forecasts and possibility intervals that closely align with observed values in most cases. According to Table 1 and Figure 1, the prediction intervals based on the crisp forecasts contains all the out-sample observed values like those computed based on the actual values [21]. In addition, the narrowness of the out-sample intervals of possibility forecasts highlight the potential for making more reliable future decisions especially when a steady time series data is dealt with. However, sensitivity of FARIMA model to the fluctuation of data should be noted [25].

$$\hat{\hat{Y}}^{*}_{t+i} = 28.0932 + 0.4994\, \hat{Y}^{*}_{(t+i)-1} - 0.519\, \hat{Y}^{*}_{(t+i)-2}, \tag{18}$$

$$\hat{Y}^{**}_{t+i} = 28.0932 + 0.4986\, \hat{\hat{Y}}^{*}_{(t+i)-1} - 0.519\, \hat{\hat{Y}}^{*}_{(t+i)-2}. \tag{19}$$

Figure 2 shows comparison of the bounds computed based on the proposed method and observed data [21].

Table 1. NTD against USD exchange rate out-sample intervals of possibility forecasts.

Date	Tseng et al. (2001) FARIMA Bounds			Proposed Method FARIMA Bounds		
	Actual Value	**LB**	**UB**	**Crisp Forecasts**	**LB**	**UB**
05-Sep	27.54	27.53	27.55	27.53	27.54	27.56
06-Sep	27.55	27.52	27.55	27.52	27.52	27.55
07-Sep	27.54	27.53	27.55	27.53	27.53	27.55
09-Sep	27.55	27.53	27.56	27.53	27.53	27.56
10-Sep	27.54	27.53	27.56	27.53	27.54	27.56
11-Sep	27.55	27.53	27.55	27.53	27.53	27.55
12-Sep	27.54	27.53	27.55	27.53	27.53	27.55
13-Sep	27.54	27.53	27.55	27.53	27.53	27.55
14-Sep	27.54	27.53	27.55	27.53	27.53	27.55
16-Sep	27.55	27.53	27.55	27.53	27.53	27.55

Furthermore, out-sample possibility interval forecasts performance evaluation in terms of mean absolute percentage error (MAPE) and root mean square error indicates that the proposed model provides improved predictive capability in LB prediction while approximately competing in UB prediction compared to the method in [21] and ARIMA FAC 0.5 intervals in [28]. The values of these metrics are presented in Table 2. Similarly, the performance metrics also indicates that the possibility intervals generated by the proposed method are narrower than the ARIMA (2,0,0) ninety-five percent

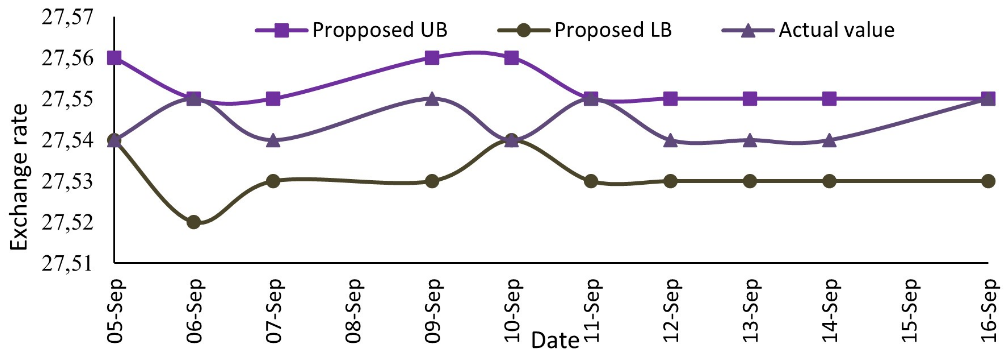

Figure 1. Crisp based NTD against USD FARIMA out-sample bounds.

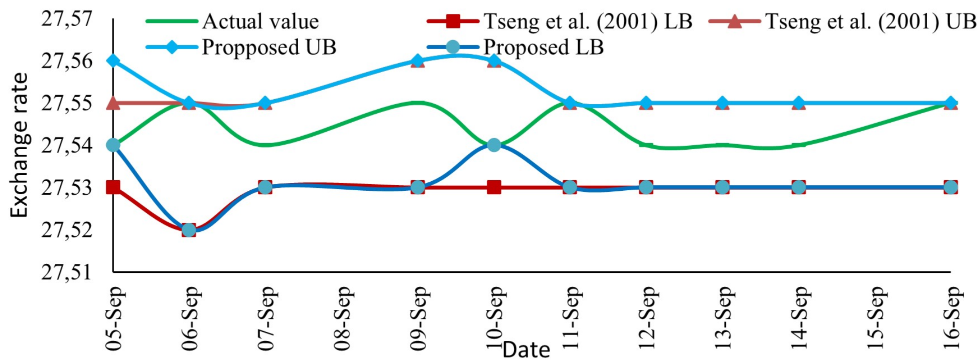

Figure 2. NTD against USD FARIMA out-sample bounds based on actual and crisp values.

confidence intervals (see Retnowardhani and Noor [28]). Thus, it can be seen that the ARIMA (2,0,0) model produced lower and upper bounds MAPE values (0.0944 & 0.0835) and RMSE values (0.0268 & 0.0239) respectively higher than those of the proposed method as presented in Table 2. Moreover, the performance evaluation clearly shows that the proposed method produces better prediction results in terms of NTD against USD forecasting.

In addition, although the crisp forecasts are generated to achieve out-sample possibility intervals forecasting, its performance evaluation metric (MAPE) is satisfactory. That is, the metric value (0.0545%) is within ten percent error. This suggests that the proposed method could be considered as an alternative out-sample point forecasting method.

Table 2. Performance evaluation results for out-sample possibility intervals.

Methods	**Tseng et al. (2001)**		**Retnowardhani & Noor (2013)**		**Proposed Method**	
Measure	**LB**	**UB**	**LB**	**UB**	**LB**	**UB**
MAPE	0.0545	0.0290	0.0472	0.0327	0.0545	0.0254
RMSE	0.0164	0.0100	0.0158	0.0114	0.0164	0.0095

3.2. Application of the Proposed Method to the Annual Mean Surface Temperature of Nigeria

The data consists of 32 sub-national (Yobe State) observations from 1991 to 2022. The first 27 observations are used as training dataset, while the last five is used for model performance evaluation. The estimated ARIMA and the Fuzzy-ARIMA models for the annual average mean surface air temperature of Nigeria data are respectively given by equations (20) and (22).

$$Y_t = 15.9184 + 0.4388\,Y_{t-1} + a_t \tag{20}$$

$$\widetilde{Y}_t = 15.9184 + (0.4388,\ 0.0322)\,Y_{t-1} + a_t \tag{21}$$

The out of sample UB prototype and crisp forecasts models are presented as equations (22) and (23). The out-sample intervals of possibility forecasts based on the observed and crisp values are presented in Table 3. It can be observed that the actual values are contained in the intervals of possibility obtained using the two set of the values. Furthermore, it can be seen from the table that the obtained intervals using the crisp values closely align with those derived with the observed values, indicating the effectiveness of the proposed method in predicting future intervals of possibility for decisions. The similarity between the two sets of intervals is further illustrated in Figure 3. The figure shows the time plot of the observed values and possibility intervals based on the proposed method.

$$\hat{\hat{Y}}^{*}_{t+i} = 15.9184 + 0.4710\,\hat{Y}^{*}_{(t+i)-1} \tag{22}$$

$$\hat{Y}^{**}_{t+i} = 15.9184 + 0.4066\,\hat{\hat{Y}}^{*}_{(t+i)-1} \tag{23}$$

Table 3. Results of FARIMA out-sample bounds forecasts for temperature data.

Year	FARIMA bounds based on observed Values			FARIMA bounds based on Proposed Method		
	Actual Value	**LB**	**UB**	**Crisp Forecasts**	**LB**	**UB**
2018	28.27	27.43297	29.25689	27.9154	27.22408	29.01492
2019	28.13	27.41264	29.23334	27.8139	27.26846	29.06633
2020	28.14	27.35572	29.16740	27.8187	27.22720	29.01852
2021	28.41	27.35979	29.17211	27.8208	27.22914	29.02077
2022	27.91	27.46956	29.29928	27.8217	27.22999	29.02176

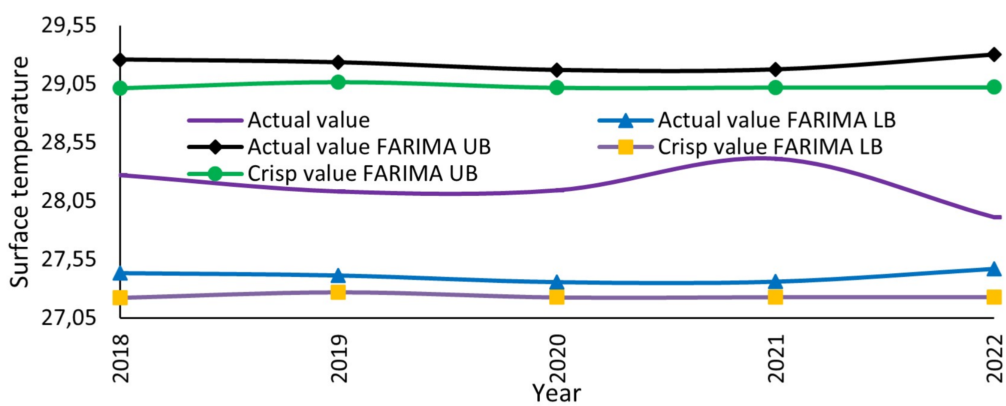

Figure 3. FARIMA out-sample bounds for temperature data.

4. Conclusions

This study addresses a gap in the literature by developing a method to generate crisp future possibility intervals from Fuzzy ARIMA bounds. The advantage of the proposed method is its unique capability to forecast future possibility intervals without relying on crisp out-of-sample observed values.

To analyze the effectiveness of the proposed method, two real datasets are used. The empirical results show that the method simultaneously achieved in-sample and out-sample intervals of possibility forecasts. In addition, the obtained experimental bounds are consistent with those obtained using the real values, indicating the effectiveness of the proposed method. Furthermore, the consistency of the results suggests that the proposed method can reliably generate future possibility intervals, especially for stable time series data. Moreover, forecasts performance evaluation in terms of mean absolute percentage error and root mean square error indicates that the proposed method provides improved predictive capability in LB predictions while approximately competing in UB predictions compared to the considered methods in the literature. This implies, the proposed model could aid policy makers in determining the worst and best projected bounds that could be used for making future decisions without relying on out-of-sample crisp observations.

Future research will explore the integration of the crisping method with metaheuristics optimization techniques to enhance forecasting accuracy and compare its performance with hybrid models.

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