



# Determination of Last Survivor Life Insurance Reserves based on the Gross Premium Valuation (GPV) Method using De'Moivre's Law

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## ABSTRACT

Insurance is one of the measures that can be used to prepare for various risks that can occur at any time. In the context of life insurance products, multiple life insurance is an efficient option because it is more economical than purchasing separate policies for two people with equivalent benefits. Unlike previous studies that focused on single life models using the GPV (Gross Premium Valuation) approach, this study develops an analysis of more complex multiple life insurance products, thereby providing a more representative picture of premium reserves for cases involving two insured parties. This study aims to formulate a mathematical model and conclude the results of prospective premium reserve calculations for last survivor whole life insurance using the GPV (Gross Premium Valuation) approach and De'Moivre's law. This study uses a quantitative method with a documentation data collection technique, namely the 2019 Mortality Table IV data published by the Indonesian Life Insurance Association (AAJI). The results of this study show that the mathematical model of premium reserves for last survivor whole life insurance using the GPV (Gross Premium Valuation) approach and De'Moivre's law is  ${}_tV^{GPV} = BA_{\overline{xy}} + U + PAG_{\overline{xy}} + A\ddot{a}_{\overline{xy}} + CA_{\overline{xy}} - G_{\overline{xy}}\ddot{a}_{\overline{xy}}$ . However, when the insured (y) dies first, the mathematical model is  ${}_tV^{GPV} = BA_{\overline{x}} + U + PAG_{\overline{xy}} + A\ddot{a}_{\overline{x}} + CA_{\overline{x}} - G_{\overline{xy}}\ddot{a}_{\overline{x}}$  while if the insured (x) dies first, the mathematical model is  ${}_tV^{GPV} = BA_{\overline{y}} + U + PAG_{\overline{xy}} + A\ddot{a}_{\overline{y}} + CA_{\overline{y}} - G_{\overline{xy}}\ddot{a}_{\overline{y}}$ . In addition, the results of the study show that there is a difference in the last survivor life insurance premium reserve between the conditions when both insured persons are still alive and when one of them dies, and that the use of De'Moivre's law results in a decreasing reserve pattern but ends up exceeding the promised benefits due to linear mortality assumptions so that the present value of the benefits does not fully decrease at the end of the coverage period. These findings indicate that the use of a uniform death distribution needs to be considered in order to produce more realistic premium reserves.

**Keywords:** Premium reserves, prospective premiums, whole life insurance, last survivor, Gross Premium Valuation (GPV), De'Moivre's law

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## 1. Introduction

Insurance is a product that provides promises or protection against unforeseeable events, with several commonly known types including health, life, education, and pension insurance [1]. In the context of life insurance, benefits are provided to heirs or entitled parties in the event of the insured's death, in accordance with the provisions of the contract [2]. Based on the duration of protection, life insurance is divided into three types: whole life insurance, term life insurance, and dual-puIDRose insurance. Based on the number of insured persons, life insurance is divided into single life insurance and group life insurance. Multiple life insurance is a contract model designed for married couples who have worked hard and want to leave a large inheritance to their children after their death [3]. Group life insurance consists of joint life and last survivor [1].

Last survivor life insurance will remain active as long as there are still insured persons alive, but joint life insurance policies will end if one of the insured persons dies [1]. In general, the premiums payable for multiple life insurance with last survivor status are lower than those for joint life status [4]. This is because multiple life insurance with last survivor status pays benefits when the last insured person dies [4]. Premium payments used for the allocation of compensation funds for the insured are calculated using annuities, with premium types such as single, annual, or monthly, and premiums are divided into gross premiums and net premiums.

Net premiums can help companies cover policy benefit obligations (claims), while gross premiums cover all company obligations, including policy benefits, costs, and profits [1]. Claims resulting from the death of a policyholder are obligations that must be fulfilled by the insurance company. The risk of claims increases at the end of the policy period because the probability of death increases with the age of the insured. Therefore, insurance companies need to assess the mortality rate of the insured actuarially in order to prepare adequate premium reserves and avoid the risk of insufficient funds for benefit payments.

Premium reserves are funds set aside to meet future obligations to policyholders. Premium reserves can be calculated prospectively or retrospectively [2]. Prospective calculation is a premium reserve that considers the present value of future expenses minus future income from each insured [2]. One method that can be used in calculating premium reserves is Gross Premium Valuation (GPV) [5].

Gross Premium Valuation (GPV) is a method that calculates premium reserves by taking gross premiums into account as a source of future income [5]. The calculation of premium reserves using the Gross Premium Valuation (GPV) method includes the total costs that must be borne by the policyholder, so that the results provided will be more in line with actual conditions [6]. In fact, insurance companies often add premiums with other additional costs charged to the insured, such as initial expenses, renewal expenses, and termination expenses [7]. The calculation of reserves using the Gross Premium Valuation (GPV) method is in accordance with Regulation of the Minister of Finance of the Republic of Indonesia Number 53/PMK.010/2012 concerning the Financial Health of Insurance Companies and Reinsurance Companies and Financial Accounting Standards Regulation (PSAK) Article 36 as amended in 2012, which stipulates that the future benefit obligations of life insurance companies to policyholders are now based on gross premiums [8].

In the study conducted by Eurico, et. al (2022), it was discussed that the Gross Premium Valuation (GPV) approach can be used to determine prospective reserves for life insurance. The study found that reserves increased each year until the policy maturity date and that premium reserves were in line with the profits promised by the company, even though the premium reserves generated in the first year were zero [9].

In addition, Prionggo also conducted research using the same method and approach, namely using the GPV method and a prospective approach. The results of the research on the single life model show that gross premium payments and benefit reserves using the GPV method and De'Moivre's law will have a greater value than the Indonesian Mortality Table IV [6]. Therefore, this study will be developed into a multiple life insurance type that is more complex than the single life model. One of the advantages of this type of life insurance is that it is more economical than purchasing separate policies for each individual by husband and wife with the same total insurance benefits [4]. This study will begin by forming a mathematical model of prospective premium reserves in last survivor whole life insurance based on the Gross Premium Valuation (GPV) method using De'Moivre's law. Next, a simulation calculation will be carried out using three case studies, followed by an intelDRretation of the analysis results.

The results of this study are expected to serve as a means to deepen understanding of actuarial concepts, particularly those related to premium reserve calculations. Thus, the results of this study are expected to contribute to helping insurance companies manage their future liabilities in a more precise, accurate, and measurable manner.

## 2. Research Methodology

This study uses secondary data obtained from the Indonesian Mortality Table IV published by the Indonesian Life Insurance Association (AAJI) at the end of 2019. The data was used as the basis for calculating the insured's mortality risk analysis. The variables used were the probability of a male aged  $x$  years, symbolized by  $(q_x)$  years, dying within the next year and the probability of a female aged  $y$  years, symbolized by  $(q_y)$ , dying within the next year.

### 2.1. Research Procedure

1. Constructing the Indonesian Mortality Table using De'Moivre's law based on the probability values in the Indonesian Mortality Table IV. The steps for constructing the table are as follows:

- (a) Determining the maximum human age based on the Indonesian Mortality Table IV.
- (b) Determining the probability that a person aged  $x$  will die at age  $x + t$  using the formula

$${}_tq_x = \frac{t}{\omega - x}, \quad 0 \leq t \leq \omega - x. \quad (1)$$

- (c) Determining the probability that a person aged  $x$  will survive to age  $x + t$  using the formula

$${}_tP_x = 1 - {}_tq_x. \quad (2)$$

2. Identifying the lifetime annuity model for last survivor whole life insurance using De'Moivre's law using the formula

$$\ddot{a}_{\bar{x}\bar{y}} = \sum_{t=0}^{\omega-z-1} v^t {}_tP_{\bar{x}\bar{y}}. \quad (3)$$

3. Determining the single premium model for last survivor whole life insurance using De'Moivre's law using the formula

$$A_{\bar{x}\bar{y}} = \sum_{t=0}^{\omega-z-1} v^{t+1} {}_t|q_{\bar{x}\bar{y}}. \quad (4)$$

4. Determining the gross premium model for last survivor whole life insurance using De'Moivre's law. The steps to determine the gross premium model are as follows.

- (a) Determining the other costs that must be borne by the insured.

- (b) Determining the cash outflow.
- (c) Determining the cash inflow.
- (d) Determine the expected loss using the formula

$$E[L] = E[PVFCO - PVFCI] = 0. \quad (5)$$

- 5. Determining the last survivor whole life insurance premium reserve model using the GPV (Gross Premium Valuation) method with De'Moivre's law using the formula [8]

$${}_tV^{GPV} = PVFCO_{(t)} - PVFCI_{(t)}. \quad (6)$$

- 6. Establishing assumptions regarding the reference interest rate, costs to be borne by the insurance company, and the amount of benefits offered by the insurance product.
- 7. By applying the model and data assumptions obtained, the Gross Premium Valuation (GPV) method is used to calculate the amount of premium reserves in the case study consisting of three assumed case examples. This is done by applying gross premiums to last survivor whole life insurance using De'Moivre's law.
- 8. Analyzing and comparing the results of the GPV (Gross Premium Valuation) method premium reserve calculations for last survivor whole life insurance using De'Moivre's law.

### 3. Result and Discussion

#### 3.1. Construction of the Indonesian Mortality Table using De'Moivre's Law

In constructing the Indonesian Mortality Table using De'Moivre's law, the parameter  $\omega$  or the maximum age of a person must first be determined [10] to be 111 years. From this knowledge, it can be determined that:

- 1. The probability of a person aged  $x$  that dying at age  $x + t$ . Using equation (1) and knowing that  $\omega = 111$ , the probability of a person aged 0 dying in the following year is

$$q_0 = \frac{1}{111 - 0}$$

$$q_0 = 0,009009009.$$

This calculation is performed until  $x = \omega$  to obtain the value of  $q_x$ , which is then included in the table of results constructed from the Indonesian Mortality Table using De'Moivre's law.

- 2. The probability that a person aged  $x$  will still be alive at age  $x + t$ . From equation (2), the probability that a person aged 0 will survive to the next age of 1 can be determined as

$$p_0 = 1 - {}_1q_0$$

$$p_0 = 1 - 0,009009009$$

$$= 0,990990991.$$

This calculation is performed until  $x = \omega$  to obtain the value of  $p_x$ , which is then included in the construction results table of the Indonesian Mortality Table using De'Moivre's law, which will subsequently be used to determine the premium reserve using De'Moivre's law.

### 3.2. Determination of the Present Value of a Life Annuity

Based on equation (3), the Actuarial Present Value (APV) of a last survivor lifetime annuity with De'Moivre's law is

$$\begin{aligned}\ddot{a}_{\bar{x}\bar{y}} &= \sum_{t=0}^{\omega-z-1} v^t {}_t p_{\bar{x}\bar{y}} \\ &= \sum_{t=0}^{\omega-z-1} v^t ({}_t p_x + {}_t p_y - {}_t p_x {}_t p_y) \\ &= \ddot{a}_x + \ddot{a}_y - \ddot{a}_{xy}\end{aligned}\tag{7}$$

where  $z \in \{x, y\}$ .

### 3.3. Determination of Single Premium for Whole Life Insurance

Using the De'Moivre's law, the single net premium for last survivor whole life insurance can be obtained based on the single premium formula for last survivor whole life insurance in equation (4)

$$\begin{aligned}A_{\bar{x}\bar{y}} &= \sum_{t=0}^{\omega-z-1} v^{t+1} {}_t q_{\bar{x}\bar{y}} \\ &= \sum_{t=0}^{\omega-z-1} v^{t+1} ({}_t q_x + {}_t q_y - {}_t q_{xy}) \\ &= A_x + A_y - A_{xy}\end{aligned}\tag{8}$$

where  $z \in \{x, y\}$ .

### 3.4. Determination of Gross Premiums for Whole Life Insurance

1. Determination of other costs to be borne by the insured The costs used in this study include three categories of existing costs, namely initial expenses in the form of underwriting costs and premium allocation costs. Then there are policy renewal expenses in the form of administrative costs and termination expenses in the form of policy redemption or claim costs. From all the costs that must be paid by the insured, the insurance company promises benefits.
2. Determination of cash outflow

$$PVFCO = \begin{cases} B_V^{(t(x)+1; t(y)+1)} + U + PA G_{\bar{x}\bar{y}} \\ + A \ddot{a}_{\overline{(t(x)+1; t(y)+1)}} + C V^{(t(x)+1; t(y)+1)} & ; t(x) = 0, 1, \dots, 111 \\ 0 & ; \text{for other } t(x) \end{cases}$$

where:

- $B$  : insurance benefit costs
- $U$  : underwriting costs
- $PA$  : premium allocation costs
- $A$  : administrative costs
- $C$  : claim costs
- $G$  : gross premium
- $t(x)$  : floor function value of the random variable of the remaining life expectancy of a man
- $t(y)$  : floor function value of the random variable of the remaining life expectancy of a woman

## 3. Determination of cash inflow

$$PVFCI = \begin{cases} G_{\overline{xy}} \ddot{a}_{\overline{(t(x)+1; t(y)+1)}}; & t(x) = 0, 1, \dots, 111 \\ 0 & ; \text{for other } t(x) \end{cases}$$

## 4. Determination of expected loss

## (a) Expected cash outflow

$$E [PVFCO] = BA_{\overline{xy}} + U + PAG_{\overline{xy}} + A\ddot{a}_{\overline{xy}} + CA_{\overline{xy}}.$$

## (b) Expected cash inflow

$$E [PVFCI] = G_{\overline{xy}} \ddot{a}_{\overline{xy}}.$$

## (c) Expected loss Using equation (5), the gross premium value for last survivor whole life insurance with De'Moivre's law can be obtained as follows

$$\begin{aligned} E [L] &= 0 \\ E [PVFCO - PVFCI] &= 0 \\ BA_{\overline{xy}} + U + PAG_{\overline{xy}} + A\ddot{a}_{\overline{xy}} + CA_{\overline{xy}} - G_{\overline{xy}} \ddot{a}_{\overline{xy}} &= 0 \\ BA_{\overline{xy}} + CA_{\overline{xy}} + A\ddot{a}_{\overline{xy}} + U + PAG_{\overline{xy}} - G_{\overline{xy}} \ddot{a}_{\overline{xy}} &= 0 \\ (B + C)A_{\overline{xy}} + A\ddot{a}_{\overline{xy}} + U - G_{\overline{xy}}(\ddot{a}_{\overline{xy}} - PA) &= 0 \\ G_{\overline{xy}} &= \frac{(B + C)A_{\overline{xy}} + A\ddot{a}_{\overline{xy}} + U}{(\ddot{a}_{\overline{xy}} - PA)}. \end{aligned} \quad (9)$$

with the values of  $\ddot{a}_{\overline{xy}}$  and  $A_{\overline{xy}}$  were determined by equations (7) and (8).

## 3.5. Determination of Premium Reserves Using the Gross Premium Valuation (GPV) Method

Based on equation (6), the formula for calculating premium reserves for whole life insurance using the Gross Premium Valuation (GPV) method is

$${}_tV^{GPV} = PVFCO_{(t)} - PVFCI_{(t)} = BA_{\overline{xy}} + U + PAG_{\overline{xy}} + A\ddot{a}_{\overline{xy}} + CA_{\overline{xy}} - G_{\overline{xy}} \ddot{a}_{\overline{xy}}. \quad (10)$$

In the case of last survivor life insurance, where the contract will continue as long as there are still insured persons alive, so that when one of the insured persons dies, the premium reserve calculation utilizes the single premium and the cash value of the annuity of the surviving insured persons. Because there is a possibility that one of the policyholders will die first, the premium reserve when the insured person (y) dies can be obtained using the formula

$${}_tV^{GPV} = BA_{\overline{x}} + U + PAG_{\overline{xy}} + A\ddot{a}_{\overline{x}} + CA_{\overline{x}} - G_{\overline{xy}} \ddot{a}_{\overline{x}}. \quad (11)$$

Conversely, if the insured (x) dies first, the premium reserve can be obtained by

$${}_tV^{GPV} = BA_{\overline{y}} + U + PAG_{\overline{xy}} + A\ddot{a}_{\overline{y}} + CA_{\overline{y}} - G_{\overline{xy}} \ddot{a}_{\overline{y}}. \quad (12)$$

## 3.6. Determination of Assumption Data

The 2019 Indonesian Mortality Table IV data, which has been constructed using De'Moivre's Law, will be used in calculating the premium reserve in three case studies. In addition, this study also makes several assumptions:

1. The reference interest rate used is 6%.
2. The costs used in this study include three categories of costs, namely initial expenses in the form of underwriting costs and premium allocation costs. Then there are renewal expenses in the form of administrative costs and termination expenses in the form of policy redemption or claim costs.
3. When the insured dies, the benefit will be paid at the end of the year.

### 3.7. Case Study

A married couple are the insured parties who will purchase an insurance product issued by PT Asuransi Allianz Life Indonesia, namely a whole life insurance product for the last survivor, with the husband aged 33 and the wife aged 29 [11]. The insurance product offers benefits of IDR 1.000.000.000. In addition, this insurance product has costs that include initial expenses in the form of underwriting costs of 15% of the net premium and premium allocation costs of 50%. Then there are policy renewal expenses in the form of administrative costs of 10% of the net premium and termination expenses in the form of policy redemption or claim costs of 25% of the net premium. There are conditions assumed to be included in the policy.

1. In the event of the first death of one of the insured, a benefit will be paid in an amount adjusted to the length of the insured's participation in the policy, without any premium refund.
2. If the premium payment period is not fulfilled, there will be a premium penalty in proportion to the coverage period that the insured failed to fulfill.

With the issuance of this insurance product, PT Asuransi Allianz Life Indonesia needs to calculate the premium reserve when

1. The insured purchases the insurance product and both parties follow the policy until the payment period is complete.
2. The insured purchases the insurance product, then in the 10th year, the wife dies.
3. The insured purchases the insurance product, then in the 10th year, the husband dies.

### 3.8. Premium Reserve Calculation in Three Case Studies Using the Gross Premium Valuation (GPV) Method

1. Calculation for the first case

This policy involves two insured parties, namely a 33-year-old husband and a 29-year-old wife. In the first case, it is assumed that both insured parties will remain alive until the payment period is complete. This policy is a last survivor whole life insurance policy where the contract will continue as long as there are still insured parties alive. Therefore, the policy coverage period is determined by the maximum lifespan of both insured parties. Using De'Moivre's law of probability and an age limit of  $\omega = 111$ , the 33-year-old husband has a maximum remaining life expectancy of 77 years, while the 29-year-old wife has a maximum remaining life expectancy of 81 years. Thus, the policy coverage period is 81 years, which is the maximum remaining life expectancy of the youngest insured.

The steps for calculating the premium reserve are as follows.

- (a) Determine the cash outflow

$$PVFCO = \begin{cases} 1,000,000,000 v^{34;30} + 15\%P_{\overline{33;29}} + 50\%G_{\overline{33;29}} \\ \quad + (10\%P_{\overline{33;29}})\ddot{a}_{\overline{34;30}} + (25\%P_{\overline{33;29}})v^{34;30} & ; 0 \leq t(x) \leq 81 \\ 0 & ; \text{for other } t(x) \end{cases}$$

(b) Determine the cash inflow

$$PVFCI = \begin{cases} G_{33;29} \ddot{a}_{34;30} & ; 0 \leq t(x) \leq 81 \\ 0 & ; \text{for other } t(x) \end{cases}$$

(c) Determine the loss

i. Expected cash outflow

$$E [PVFCO] = 1,000,000,000A_{33;29} + 15\%P_{33;29} + 50\%G_{33;29} + \left(10\%P_{33;29}\right) \ddot{a}_{33;29} + \left(25\%P_{33;29}\right) A_{33;29}.$$

ii. Expected cash inflow

$$E [PVFCI] = G_{33;29}\ddot{a}_{33;29}.$$

iii. Expected loss

Using equation (5), the gross premium value for whole life insurance with the De’Moivre law can be obtained as follows

$$\begin{aligned} 0 &= E [L] \\ 0 &= E [PVFCO - PVFCI] \\ 0 &= 1,000,000,000A_{33;29} + 15\%P_{33;29} + 50\%G_{33;29} + \left(10\%P_{33;29}\right) \ddot{a}_{33;29} \\ &\quad + \left(25\%P_{33;29}\right) A_{33;29} - G_{33;29}\ddot{a}_{33;29} \\ G_{33;29} &= \frac{\left(1,000,000,000 + \left(25\%P_{33;29}\right)\right) A_{33;29} + \left(10\%P_{33;29}\right) \ddot{a}_{33;29} + \left(15\%P_{33;29}\right)}{\left(50\% - \ddot{a}_{33;29}\right)}. \end{aligned} \tag{13}$$

Based on equation (10), the determination of the premium reserve for this first case can be solved by

$$\begin{aligned} {}_tV^{GPV} &= \left(1,000,000,000A_{33;29} + 15\%P_{33;29} + 50\%G_{33;29} + \left(10\%P_{33;29}\right) \ddot{a}_{33;29} \right. \\ &\quad \left. + \left(25\%P_{33;29}\right) A_{33;29}\right) - G_{33;29}\ddot{a}_{33;29} \end{aligned} \tag{14}$$

where

$$P_{\bar{x}\bar{y}} = \frac{A_{\bar{x}\bar{y}}}{\ddot{a}_{\bar{x}\bar{y}}} \tag{15}$$

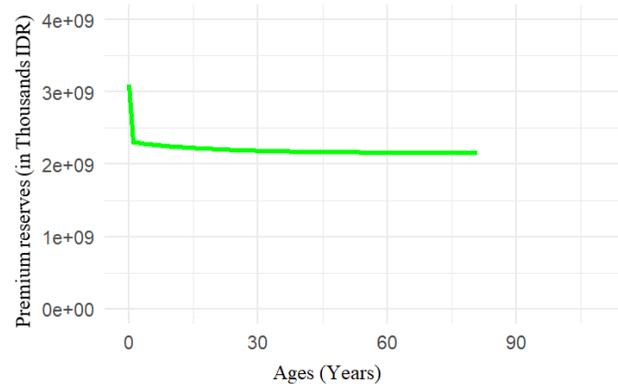
for  $\ddot{a}_{33;29}$ ,  $A_{33;29}$  and  $G_{33;29}$  using equations (7), (8), and (13).

Based on calculations, the net premium ( $P_{33;29}$ ) is IDR4,631,138,377 and the gross premium ( $G_{33;29}$ ) is IDR14,116,040.250. Thus, using equation (14), the premium reserve in the first case is presented in the following.

**Table 1.** Calculation Results of the GPV Premium Reserve Amount in the First Case

$t$	$BA_{33;29}$	$U$	$PAG_{33;29}$	$A\ddot{a}_{33;29}$	$CA_{33;29}$	$G_{33;29}\ddot{a}_{33;29}$	$PVFCO - PVFCI$
0	IDR943,396,226	IDR694,670,757	0.5	IDR463,113,838	IDR1,092,249,617	IDR187,932,538	IDR3,099,464,169
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
11	IDR970,181,960	0	0	IDR243,961,598	IDR1,123,261,727	IDR93,997,936	IDR2,24,407,348
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
81	IDR1.000.000.000	0	0	0	IDR1,157,784,594	0	IDR2,157,784,594

Based on Table 1, in year 0 or the first year of the insurance policy, the premium reserve generated was IDR3,099,464,169. From year 1 to year 77, which is the last premium payment period, the premium reserve continued to decline. This can occur because the calculation of premium reserves in this case uses De'Moivre's law of probability of death, where the probability of survival decreases constantly and the survival probability value becomes relatively simple. Due to its linear mortality structure, the present value of benefits for last survivor contracts decreases more rapidly as the insured ages. As a result, the present value of benefits does not fully decrease to zero at the end of the coverage period, while future premiums no longer exist. In prospective reserve calculations, the reserve in the last year is equal to the remaining present value of benefits, causing the premium reserve in the last year to exceed the promised benefits. The graph of the GPV premium reserve in the second case is as follows.



**Figure 1.** GPV Premium Reserve Graph for the First Case

Figure 1 shows the decline in premium reserves in the second case during the policy period. In the first year, i.e., year 0, there is a drastic decrease to year 1. This is because at the beginning of the contract, the present value of future premiums is still very large, thereby reducing the amount of reserves. Entering the second year, part of the premium has been paid, so the present value of future premiums drops dramatically, while the present value of benefits only decreases slightly. In addition, there is a drastic decrease in premium reserves when the policy moves from year 10 to year 11. This is because it is assumed that the policy contains a condition that if the first death occurs to one of the insured, a benefit will be paid in an amount adjusted to the length of the insured's participation in the policy. This policy also assumes that there is a premium penalty clause if the premium payment period is not fulfilled, where the amount of the penalty is determined based on the proportion of the coverage period that was not fulfilled by the insured who died first.

2. Calculation for the second case

Pada In the second case, it is assumed that the wife dies in the 10th year. Therefore, starting in the 11th year, the policy only covers the husband, who was originally 33 years old. As a result, there is a change in the policy coverage period, which was originally until the age of 81, but is now changed to only 77 years, which is the maximum remaining age of the insured who is still alive. This is because the policy is a last survivor whole life insurance policy, where the contract will continue even if one of the insured persons dies. Therefore, if one of the insured persons dies, the policy coverage period changes to the remaining maximum age of the surviving insured person.

The steps for calculating the premium reserve are as follows.

(a) Determine the cash outflow

$$PVFCO = \begin{cases} 1,000,000,000 v^{34;30} + 15\%P_{33;29} + 50\%G_{33;29} \\ \quad + (10\%P_{33;29})\ddot{a}_{34;30} + (25\%P_{33;29})v^{34;30} & ; 0 \leq t(x) \leq 10 \\ 830,000,000 v^{34} + 15\%P_{33;29} + 50\%G_{33;29} \\ \quad + (10\%P_{33;29})\ddot{a}_{34} + (25\%P_{33;29})v^{34} & ; 11 \leq t(x) \leq 77 \\ 0 & ; \text{for other } t(x) \end{cases}$$

The policy assumes that if the first death occurs to one of the insured, a benefit will be paid in an amount adjusted to the length of the deceased insured's participation in the policy. Thus, the company must pay a benefit of 12% of the total promised benefit because the wife had participated in the policy for 10 years of the total agreed coverage period, which was 81 years. This policy also assumes a premium penalty clause if the premium payment period is not fulfilled, where the amount of the deduction is determined based on the proportion of the coverage period that was not fulfilled by the insured who died first. Because the wife died early, the coverage period, which was originally planned for 81 years, was only realized until the 77th year. Thus, there was a 4-year period of coverage that was not fulfilled, resulting in a premium penalty of 5%. As a result, the company is only obligated to pay 83% of the total promised benefits in the event of the death of the last insured, namely the husband. Therefore, from the 11th year to the 77th year, the benefit is only IDR830,000,000, which is 83% of IDR1,000,000,000.

(b) Determining cash inflows

$$PVFCI = \begin{cases} G_{33;29}\ddot{a}_{34;30} & ; 0 \leq t(x) \leq 10 \\ G_{33;29}\ddot{a}_{34} & ; 11 \leq t(x) \leq 77 \\ 0 & ; \text{for others } t(x) \end{cases}$$

(c) Determining loss

i. Expected cash outflow

$$E[PVFCO] = \begin{cases} 1,000,000,000 A_{33;29} + 15\%P_{33;29} + 50\%G_{33;29} \\ \quad + (10\%P_{33;29})\ddot{a}_{33;29} + (25\%P_{33;29})A_{33;29} & ; 0 \leq t(x) \leq 10 \\ 830,000,000 A_{33} + 15\%P_{33;29} + 50\%G_{33;29} \\ \quad + (10\%P_{33;29})\ddot{a}_{33} + (25\%P_{33;29})A_{33} & ; 11 \leq t(x) \leq 77 \\ 0 & ; \text{other } t(x) \end{cases}$$

ii. Expected cash inflow

$$E[PVFCI] = \begin{cases} G_{33;29}\ddot{a}_{33;29} & ; 0 \leq t(x) \leq 10 \\ G_{33;29}\ddot{a}_{33} & ; 11 \leq t(x) \leq 77 \\ 0 & ; \text{for other } t(x) \end{cases}$$

iii. Expected loss

Expected loss for  $0 \leq t(x) \leq 10$  and  $11 \leq t(x) \leq 77$  can use equation (13) because this insurance product is last survivor life insurance. Therefore, the expected loss calculated to obtain the gross premium equation uses the combined expected loss in equation (13).

The premium reserve for  $0 \leq t(x) \leq 10$  can be determined using equation (14), while for  $11 \leq t(x) \leq 77$  it can be solved as follows

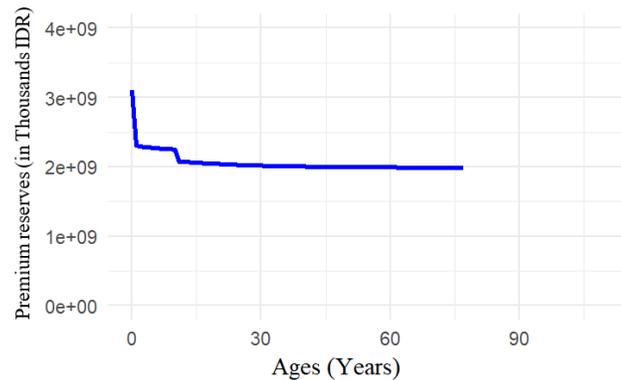
$$\begin{aligned}
 {}_tV^{GPV} = & (1,000,000,000A_{33} + (15\% \times P_{33;29}) + 50\%G_{33;29} + (10\% \times P_{33;29})\ddot{a}_{33} \\
 & + (25\% \times P_{33;29})A_{33}) - G_{33;29}\ddot{a}_{33}.
 \end{aligned}
 \tag{16}$$

with  $G_{33;29}$  using equation (13). Based on calculations using Rstudio software, the net premium ( $P_{33;29}$ ) is IDR4,631,138,377 and the gross premium ( $G_{33;29}$ ) is IDR13,444,181,772. Thus, using equations (14) and (16), the premium reserve in the first case is presented in the Table 2.

**Table 2.** Calculation Results of the GPV Premium Reserve Amount in the Second Case

$t$	PVFCO			PVFCI			$V^{GPV}$
	$BA_{33;29}$	$U$	$PAG_{33;29}$	$A\ddot{a}_{33;29}$	$CA_{33;29}$	$G_{33;29}\ddot{a}_{33;29}$	PVFCO - PVFCI
0	IDR943,396,226	IDR694,670,757	0.5	IDR463,113,838	IDR1,092,249,617	IDR187,932,538	IDR3,099,464,169
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
11	IDR805,303,696	0	0	IDR243,442,417	IDR1,123,335,196	IDR93,797,897	IDR2,078,283,411
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
77	IDR830,000,000	0	0	0	IDR1,157,784,594	0	IDR1,987,784,594

Based on Table 2, in year 0 or the first year of the insurance policy, the premium reserve generated was IDR3,099,464,169. From year 1 to year 77, which is the last premium payment period, the premium reserve continued to decline. This can occur because the calculation of premium reserves in this case uses De’Moivre’s law of probability of death, where the probability of survival decreases constantly and the survival probability value becomes relatively simple. Due to its linear mortality structure, the present value of benefits for last survivor contracts decreases more rapidly as the insured ages. As a result, the present value of benefits does not fully decrease to zero at the end of the coverage period, while future premiums no longer exist. In prospective reserve calculations, the reserve in the last year is equal to the remaining present value of benefits, causing the premium reserve in the last year to exceed the promised benefits. The graph of the GPV premium reserve in the second case is as follows:



**Figure 2.** GPV Premium Reserve Graph for the Second Case

Figure 3 shows the decline in premium reserves in the second case during the policy period. In the first year, i.e., year 0, there is a drastic decrease to year 1. This is because at the beginning of the contract, the present value of future premiums is still very large, thereby

reducing the amount of reserves. Entering the second year, part of the premium has been paid, so the present value of future premiums drops dramatically, while the present value of benefits only decreases slightly. In addition, there is a drastic decrease in premium reserves when the policy moves from year 10 to year 11. This is because it is assumed that the policy contains a condition that if the first death occurs to one of the insured, a benefit will be paid in an amount adjusted to the length of the insured's participation in the policy. This policy also assumes that there is a premium penalty clause if the premium payment period is not fulfilled, where the amount of the penalty is determined based on the proportion of the coverage period that was not fulfilled by the insured who died first.

### 3. Calculation for the third case

In the second case, it is assumed that the husband dies in the 10th year. Therefore, starting in the 11th year, the policy only covers the wife, who was initially 29 years old. Therefore, the policy coverage period remains 81 years, the same as in the first case but different from the second case. This is because 81 years is the maximum remaining age of the insured who is still alive, namely the wife. The steps for calculating the premium reserve are as follows:

(a) Determine the cash outflow

$$PVFCO = \begin{cases} 1,000,000,000v^{34;30} + (15\%P_{33;29}) + 50\%G_{33;29} & ; 0 \leq t(x) \leq 10 \\ + (10\%P_{33;29})\ddot{a}_{34;30} + (25\%P_{33;29})v^{34;30}; & \\ 880,000,000v^{30} + (15\%P_{33;29}) + 50\%G_{33;29} & ; 11 \leq t(x) \leq 81 \\ + (10\%P_{33;29})\ddot{a}_{30} + (25\%P_{33;29})v^{30}; & \\ 0; & ; \text{others } t(x) \end{cases} .$$

The policy assumes that if the first death occurs to one of the insured parties, a benefit will be paid in an amount adjusted to the length of the deceased insured party's participation in the policy. Thus, the company must pay a benefit of 12% of the total promised benefit because the husband had participated in the policy for 10 years of the total agreed coverage period, which is 81 years. This policy also assumes that there is a premium penalty clause if the premium payment period is not fulfilled, where the amount of the deduction is determined based on the proportion of the coverage period that was not fulfilled by the insured who died first. However, because there was no change in the policy coverage period, the premium penalty did not apply. Therefore, the company was obliged to pay benefits amounting to 88% of the total promised benefits in the event of the death of the last insured, namely the wife. Therefore, from the 11th to the 77th year, the remaining benefit is IDR880.000.000 , which is 88% of IDR1,000,000,000.

(b) Determining cash inflow

$$PVFCI = \begin{cases} G_{33;29} \ddot{a}_{34;30} & ; 0 \leq t(x) \leq 10 \\ G_{33;29} \ddot{a}_{30} & ; 11 \leq t(x) \leq 81 \\ 0 & ; \text{for other } t(x) \end{cases}$$

(c) Determining loss

i. Expected cash outflow

$$E[PVFCO] = \begin{cases} 1,000,000,000A_{33;29} + (15\% \times P_{33;29}) + 50\%G_{33;29} & ; 0 \leq t(x) \leq 10 \\ \quad + (10\% \times P_{33;29})\ddot{a}_{33;29} + (25\% \times P_{33;29})A_{33;29}, & \\ 880,000,000A_{29} + (15\% \times P_{33;29}) + 50\%G_{33;29} & ; 11 \leq t(x) \leq 81 \\ \quad + (10\% \times P_{33;29})\ddot{a}_{29} + (25\% \times P_{33;29})A_{29}, & \\ 0, & ; \text{others } t(x) \end{cases}$$

ii. Expected cash inflow

$$E[PVFCI] = \begin{cases} G_{33;29}\ddot{a}_{33;29} & ; 0 \leq t(x) \leq 10 \\ G_{33;29}\ddot{a}_{29} & ; 11 \leq t(x) \leq 81 \\ 0 & ; \text{for other } t(x) \end{cases}$$

iii. Expected loss Expected loss for  $0 \leq t(x) \leq 10$  and  $11 \leq t(x) \leq 77$  can use equation (13) because this insurance product is last survivor life insurance. Therefore, the expected loss calculated to obtain the gross premium equation uses the combined expected loss in equation (13).

The determination of the premium reserve for  $0 \leq t(x) \leq 10$  can use equation (14), while for  $11 \leq t(x) \leq 77$  it can be solved by

$${}_tV^{GPV} = (1,000,000,000A_{29} + (15\% \times P_{33;29}) + 50\%G_{33;29} + (10\% \times P_{33;29})\dot{a}_{29} + (25\% \times P_{33;29})A_{29}) - G_{33;29}\dot{a}_{29} \tag{17}$$

with  $G_{33;29}$  using equation (13).

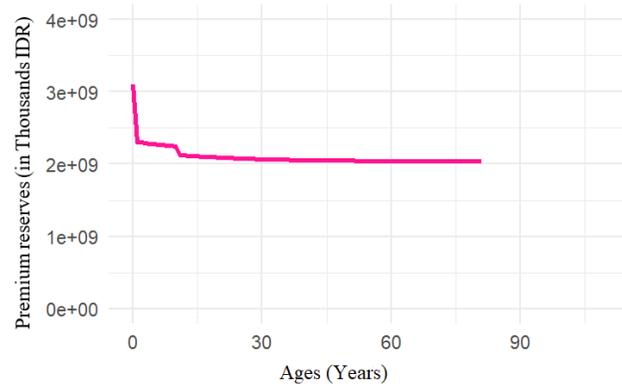
Based on calculations, the net premium is IDR4,631,138,377 and the gross premium  $G_{33;29}$  is IDR14,116,040,250. Therefore, using equations (14) and (16), the premium reserve in the third case is presented as follows.

**Table 3.** Calculation Results of the GPV Premium Reserve Amount for the Third Case

<i>t</i>	<i>PVFCO</i>			<i>PVFCI</i>			<i>V<sup>GPV</sup></i>
	<i>BA<sub>33;29</sub></i>	<i>U</i>	<i>PAG<sub>33;29</sub></i>	<i>A<math>\ddot{a}</math><sub>33;29</sub></i>	<i>CA<sub>33;29</sub></i>	<i>G<sub>33;29</sub><math>\dot{a}</math><sub>33;29</sub></i>	<i>PVFCO – PVFCI</i>
0	IDR 943,396,226	IDR 694,670,757	0.5	IDR 463,113,838	IDR 1,092,249,617	IDR 187,932,538	IDR 3,099,464,169
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
11	IDR 853,810,207	0	0	IDR 243,495,963	IDR 1,123,327,617	IDR 93,818,528	IDR 2,126,815,260
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
81	IDR 880,000,000	0	0	0	IDR 1,157,784,594	0	IDR 2,037,784,594

Based on Table 3, in year 0 or the first year of the insurance policy, the premium reserve generated is IDR3,099,464,169. From year 1 to year 81, which is the last premium payment period, the premium reserve continues to decline. This can occur because the premium reserve calculation in this case uses De’Moivre’s law of mortality, where the probability of survival decreases constantly and the survival probability value becomes relatively simple. Because the mortality structure is linear, the present value of benefits for the last survivor contract decreases more rapidly as the insured ages. As a result, the present value of benefits does not fully decrease to zero at the end of the coverage period, while future premiums no longer exist. In prospective reserve calculations, the reserve in the last year is equal to the remaining present value of benefits,

causing the premium reserve in the last year to exceed the promised benefits. The graph of the GPV premium reserve in the second case is as follows:



**Figure 3.** GPV Premium Reserve Graph for the Third Case

Figure 3 shows a decline in premium reserves in the third case during the policy period. In the first year, i.e., year 0, there is a drastic decline to year 1. This is because at the beginning of the contract, the present value of future premiums is still very large, thereby reducing the amount of reserves. Entering the second year, part of the premium has been paid, so the present value of future premiums declines drastically, while the present value of benefits only decreases slightly. In addition, there is a drastic decrease in the premium reserve when the policy moves from year 10 to year 11. This is because it is assumed that the policy has a condition that if the first death occurs to one of the insured, the benefit will be paid in an amount adjusted to the length of the insured's participation period in the policy. This policy also assumes that there is a premium penalty clause if the premium payment period is not fulfilled, where the amount of the penalty is determined based on the proportion of the coverage period that was not fulfilled by the insured who died first. However, in this third case, no premium penalty is imposed because there is no change in the policy coverage period.

### 3.9. Comparing the Results of Premium Reserve Calculations in Three Case

**Table 4.** Calculation Results of the  $V^{GPV}$  Premium Reserve Amount

$t$	$V^{GPV}$		
	First Case	Second Case	Third Case
0	IDR3,099,464,169	IDR3,099,464,169	IDR3,099,464,169
1	IDR2,303,235,890	IDR2,303,235,890	IDR2,303,235,890
2	IDR2,295,956,451	IDR2,295,956,451	IDR2,295,956,451
3	IDR2,288,984,143	IDR2,288,984,143	IDR2,288,984,143
4	IDR2,282,313,121	IDR2,282,313,121	IDR2,282,313,121
5	IDR2,275,936,602	IDR2,275,936,602	IDR2,275,936,602
6	IDR2,269,847,056	IDR2,269,847,056	IDR2,269,847,056
7	IDR2,264,036,377	IDR2,264,036,377	IDR2,264,036,377
8	IDR2,258,496,018	IDR2,258,496,018	IDR2,258,496,018
9	IDR2,253,217,123	IDR2,253,217,123	IDR2,253,217,123
10	IDR2,248,190,267	IDR2,248,190,267	IDR2,248,190,267
11	IDR2,243,407,348	IDR2,078,283,411	IDR2,126,815,260
⋮	⋮	⋮	⋮

$t$	$V^{GPV}$		
	First Case	Second Case	Third Case
77	IDR2,159,361,215	IDR1,987,784,594	IDR2,039,423,144
78	IDR2,159,179,182	0	IDR2,039,233,954
79	IDR2,158,954,236	0	IDR2,039,000,167
80	IDR2,158,612,361	0	IDR2,038,644,859
81	IDR2,157,784,594	0	IDR2,037,784,594

Based on Tabel 4, it can be seen that the premium reserves from year 0 to year 10 have the same results. This is because both insured parties are still alive, therefore the calculation of premium reserves utilizes single premiums and estimated annuity cash values for both insured parties. Then, in year 11, the premium reserves from the three cases begin to have different results. This is because there is an assumption that one of the insured persons who participated in the insurance product contract died in year 10. Therefore, the calculation of the premium reserve is carried out using a single premium and the estimated annuity cash value for the surviving insured person.

Based on the study, it can be seen that starting in the 11th year, the second case generated the least premium reserve. This is because the second case had fewer remaining contracts than the first and third cases. Even though the first and third cases had the same number of remaining contracts, the premium reserves generated were still different. The premium reserve in the third case is smaller than the premium reserve generated by the first case. This is because in the third case, a death claim occurred in the 10th year. Meanwhile, in the first case, both insured persons were still alive until the end of the policy, so there were no death claims during the policy period.

Based on Tabel 4, it can be seen that the premium reserves generated in the last year show different results. This is due to the assumption that when one of the insured dies, the benefit will be paid in an amount adjusted to the length of the insured’s participation in the policy. Among the three cases, the second case recorded the smallest premium reserve in the last year of the policy. This is because in this case, the coverage period did not match the coverage period agreed upon at the beginning of the policy contract. Therefore, in the second case, there was a premium penalty. The comparative graph of the GPV premium reserve in the third case is as follows:

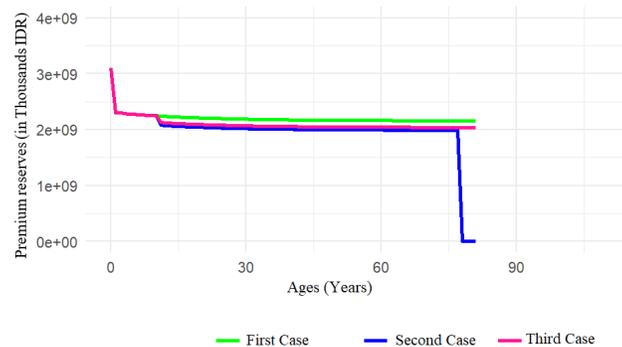


Figure 4. Comparison Chart of the Three GPV Premium Reserve Cases

Based on Figure 4, it can be seen that in the period before the death of one of the insured, the amount of premium reserves generated was the same. However, when entering the 11th year, where it is assumed that one of the insured died, the premium reserves generated would decrease significantly. This is because there is a death claim in an amount adjusted to the length of the insured’s participation

in the policy, and the policy begins to cover only one insured, so the calculation of the premium reserve utilizes a single premium and the estimated annuity cash value for the surviving insured. Of the three cases, the smallest premium reserve is the second case, where the death of the wife means that the policy only covers the husband until the end of the payment period or until his death.

#### 4. Conclusions

Based on the results of the study, the following conclusions can be drawn: The mathematical model for the last survivor whole life insurance premium reserve using the Gross Premium Valuation (GPV) method is as follows.

1. The mathematical model for the last survivor whole life insurance premium reserve using the Gross Premium Valuation (GPV) method is

$${}_tV^{GPV} = BA_{\overline{xy}} + U + PAG_{\overline{xy}} + A\ddot{a}_{\overline{xy}} + CA_{\overline{xy}} - G_{\overline{xy}}\ddot{a}_{\overline{xy}}.$$

However, when the insured (y) dies first, the mathematical model for the last survivor whole life insurance premium reserve using the Gross Premium Valuation (GPV) method is

$${}_tV^{GPV} = BA_{\overline{x}} + U + PAG_{\overline{xy}} + A\ddot{a}_{\overline{x}} + CA_{\overline{x}} - G_{\overline{xy}}\ddot{a}_{\overline{x}}.$$

Conversely, if the insured (x) dies first, the mathematical model for the last survivor whole life insurance premium reserve using the Gross Premium Valuation (GPV) method is

$${}_tV^{GPV} = BA_{\overline{y}} + U + PAG_{\overline{xy}} + A\ddot{a}_{\overline{y}} + CA_{\overline{y}} - G_{\overline{xy}}\ddot{a}_{\overline{y}}.$$

2. Based on the results of the calculations in the three cases studied, it can be concluded that there is a difference in the last survivor life insurance premium reserve when both insured persons are still alive and when one of the insured persons has died. This is because when one of the insured persons dies, the premium reserve calculation uses a single premium and the estimated cash value of the annuity only for the insured person who is still alive. In addition, based on the research, it can be concluded that the calculation of premium reserves using De'Moivre's law of mortality results in premium reserves that decrease until the end of the policy period and results in premium reserves at the end of the policy period exceeding the promised benefits. This is because the De'Moivre mortality structure is linear, so that the present value of benefits does not fully decline to zero at the end of the coverage period, while future premiums no longer exist and in prospective premium reserve calculations, the reserve in the last year is equal to the present value of the remaining benefits, causing the premium reserve in the last year to exceed the promised benefits.

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