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Dynamics and Bifurcation of Predator-Prey Model with Type II Holling Response Function and Anti-Predator Behavior

M. Fadillah[1*], Marwan[1], Lailia Awalushaumi[1]

[1] *Department of Mathematics, Universitas Mataram, Indonesia*
[*] *Corresponding author: muhammadfadillahbom@gmail.com*

A B S T R A C T

This study aims to analyze the dynamical behavior of a predator–prey model incorporating a Holling type II functional response and an anti-predator mechanism. The methods employed include dynamical systems analysis, namely equilibrium point determination, stability analysis using the Jacobian matrix, and bifurcation analysis. This analytical approach is supported by numerical simulations to construct phase portraits and illustrate system trajectories around equilibrium points. The results reveal various complex behaviors, particularly changes in stability and the emergence of bifurcation phenomena, which are influenced by variations in key parameters such as the saturation parameter, anti-predator parameter, and predator–prey interaction rate. Specifically, three types of bifurcations are identified: Hopf, transcritical, and saddle-node bifurcations. Hopf bifurcation leads to stable periodic oscillations, transcritical bifurcation results in an exchange of stability between equilibrium points, while saddle-node bifurcation causes the appearance or disappearance of equilibrium points. Numerical simulations further support these findings by illustrating system behavior before, at, and after bifurcation. Overall, the interaction between functional response saturation and anti-predator mechanisms plays a crucial role in determining the stability and dynamics of predator–prey populations.

Keywords: predator-prey model, Holling response function, transcritical, Hopf, saddle-node.

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1. Introduction

Mathematical modeling is a mathematically represented model of a system or phenomenon that occurs in the real world. Mathematical models are usually used to make assumptions about variables, parameters, and function shapes [1]. Mathematical modeling serves as a bridge between real-world phenomena and mathematical analysis, by representing real problems in the form of mathematical models that can be analyzed systematically [2] [3]. One of the important fields of application of mathematical modeling is biology, especially in ecological studies that study the interactions between living things and their environment. One of the most basic forms of interaction in ecology is the interaction between predator and prey. Predator-prey interactions are an important feature of natural ecosystems, where predators depend on prey to survive, and prey try to avoid predation to maintain the continuation of their species. This interaction forms an ongoing dynamic cycle [4].

According to Finizio, et al. (1998) in [5] the mathematical model that describes this interaction is known as the predator-prey model. This model was first introduced by Lotka in 1925 and Volterra in 1926, so it is also known as the Lotka-Volterra model. This model is based on several assumptions, namely that the prey population will grow exponentially in the absence of predators, the predator population will decrease in the absence of prey, predators can consume an unlimited number of prey, and the environment is considered to have no complexity.

However, in ecological reality, these interactions are much more complex. Predators do not always actively hunt prey indefinitely. At some point, they may experience saturation, a condition where they no longer increase their consumption rate despite high prey populations. In addition, prey are not passive to predator threats, they develop various survival strategies, such as camouflage or fight back, known as anti-predator behavior [5]. To describe this phenomenon more realistically, a Holling type II response function is used. This function takes into account handling time, power and strength, which causes the predator's consumption rate to increase when prey are few, but slow down when prey populations are high.

The purpose of this study is to examine the dynamics and bifurcation of the predator-prey model using Holling type II response functions and anti-predator behavior, in order to better understand the complex interactions between populations and how changes in certain parameters can affect the stability of the system. Bifurcation is a qualitative change in the behavior of a dynamical system that occurs when one of the parameters in the system changes. In the context of system dynamics, bifurcation arises when a fixed point undergoes a change in behavior, such as a shift in stability or existence. In general, bifurcation describes the transition of a system from one pattern of behavior to another due to changes in the value of certain parameters [6]. This phenomenon is an important part of mathematical and physical studies, as it shows that small changes in parameters can lead to large changes in system behavior, such as the appearance of oscillations, chaotic conditions, or stability shifts [7]. Based on this, this study aims to analyze the dynamics and bifurcation of the predator-prey model with Holling type II response function and anti-predator behavior.

2. Research Methods

This study analyzes the dynamics and bifurcations that occur from mathematical models, especially predator-prey models, by changing the parameters in the model. The analysis steps are as follows:

1. Model formulation. The model used is a predator-prey model, the model is formed using certain assumptions, such as the existence of anti-predator behavior and predators do not always actively hunt prey indefinitely, when they reach a certain point, they can experience saturation, which is a condition where the predator no longer increases the consumption rate even though the prey population is high.
2 Determine the equilibrium point of the model. To obtain the equilibrium point in a system of differential equations, the first derivative in the system of differential equations is equal to zero.

Definition 2.1. [8] The point $\bar{x} \in \mathbb{R}^n$ is called an equilibrium point of system $\dot{x} = f(x)$ if $f(\bar{x}) = 0$.

The expression $\dot{x} = f(x)$ represents a dynamical system or a system of differential equations, where $\bar{x} \in \mathbb{R}^n$ is the state vector describing the system variables, $\dot{x}$ denotes the time derivative indicating the rate of change, and $f(x)$ is a function that governs the evolution of the system. Thus, the behavior of the system over time is determined by the function $f(x)$. The condition $f(\bar{x}) = 0$ represents an equilibrium point, indicating that at the point $\bar{x}$, no change occurs because all derivatives are equal to zero. Consequently, the system remains in a constant state at this point.

3. Linearization of vector field in the vicinity of each hyperbolic equilibrium. The matrix A is the Jacobian matrix of the function $f(x)$ evaluated at the equilibrium point $\bar{x}$. This matrix is denoted as $A = Df(\bar{x})$, where $Df(\bar{x})$ is the partial derivative of each component of the function $f(x)$ with respect to each variable x, as follows.

$$Df(\bar{x}) = \begin{pmatrix} \frac{\partial f_1(\bar{x})}{\partial x_1} & \frac{\partial f_1(\bar{x})}{\partial x_2} & \cdots & \frac{\partial f_1(\bar{x})}{\partial x_n} \\ \frac{\partial f_2(\bar{x})}{\partial x_1} & \frac{\partial f_2(\bar{x})}{\partial x_2} & \cdots & \frac{\partial f_2(\bar{x})}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n(\bar{x})}{\partial x_1} & \frac{\partial f_n(\bar{x})}{\partial x_2} & \cdots & \frac{\partial f_n(\bar{x})}{\partial x_n} \end{pmatrix} \tag{1}$$

4. Analysis of stability of the equilibrium. Suppose $f \in \mathbb{R}^2$ and λ is the eigenvalue of the Jacobian matrix $Df(\bar{x})$, the stability of the equilibrium point can be found in the following Table 1 [9]

Table 1. Equilibrium Point Stability Properties

Eigenvalue	Type	Stability
$\lambda_1 \geq \lambda_2 > 0$	Source	Unstable
$\lambda_1 \leq \lambda_2 > 0$	Sink	Stable
$\lambda_1 < 0 < \lambda_2$	Saddle	Unstable
$\lambda_{1,2} = r \pm i\mu$	Spiral	
$r > 0$	Spiral source	unstable
$r < 0$	Spiral sink	asymptotically stable
$r = 0$	Center	stable

with r being the real part and μ being the imaginary part of the complex eigenvalue.

Theorem 2.2. [10] If A is an $n \times n$ matrix, then λ is an eigenvalue of A if and only if λ satisfies the equation

$$\det(\lambda I - A) = 0. \tag{2}$$

This equation is called the *characteristic equation* of A.

5. Numerical bifurcation by varying some parameter values of the model.
The criteria for bifurcation are as follows.
   (a) The transcritical bifurcation that occurs in the system undergoes a transition in which one equilibrium point that was originally stable becomes unstable, while another equilibrium point, which was previously unstable, becomes stable.
   (b) A Hopf bifurcation occurs when a change in parameters causes a shift in the stability of the system, which then gives rise to periodic solutions. At this point, the system undergoes a transition from a stable state to stable periodic oscillations.

(c) A bifurcation fold or saddle-node bifurcation occurs when the parameter is changed, causing a system that initially has two equilibrium points to merge at a certain parameter value, causing both equilibrium points to disappear from the system.

3. Result and Discussion

This section presents the results of the analysis and discussion of the developed model. The primary focus is to examine the dynamical behavior of the predator–prey system through both analytical and numerical approaches. The analysis is conducted to understand the characteristics of equilibrium points, their stability, and the potential occurrence of bifurcation phenomena due to parameter variations.

3.1. Model Formulation

Suppose that in a closed region, there is a prey population and a predator population. The prey population density at time t is denoted by $x(t)$, while the predator population density at time t is denoted by $y(t)$.

The dynamics of the prey population are assumed to be influenced by two main factors, namely logistic growth and interactions with predators. Mathematically, these dynamics are expressed as

$$\dot{x}(t) = rx\left(1 - \frac{x}{K}\right) - aP(x)y. \tag{3}$$

where r represents the intrinsic growth rate of the prey, K denotes the environmental carrying capacity, and a is the interaction rate between prey and predator that affects the prey growth rate. The function $P(x)$ represents the functional response, which describes the relationship between prey density and the intensity of predation [11].

The dynamics of the predator population are influenced by three main factors, namely natural mortality, the presence of anti-predator behavior in the prey, and interactions with the prey. Mathematically, this is expressed as

$$\dot{y}(t) = bP(x)y - my - exy. \tag{4}$$

where b represents the interaction rate between prey and predator that affects the predator growth rate, m denotes the natural mortality rate of the predator, and e represents the level of anti-predator behavior [11].

used is the Holling type II response function in [11]. The Holling type II response function was chosen because it realistically represents the limitations of predators in preying due to handling time. That the following model form is obtained.

$$\begin{aligned} \dot{x}(t) &= rx\left(1 - \frac{x}{K}\right) - \frac{axy}{c+x} \\ \dot{y}(t) &= \frac{bxy}{c+x} - my - exy \end{aligned} \tag{5}$$

Table 2. Description of Variables and Parameters in Model 5

Symbol	Type	Description	Condition
x	Variable	Number of prey population	$x \in \mathbb{R}$
y	Variable	Number of predator population	$y \in \mathbb{R}$
r	Parameter	Intrinsic growth rate of prey	$r > 0$
K	Parameter	Environmental carrying capacity	$K > 0$
a	Parameter	Interaction rate between prey and predator affecting prey growth	$a \geq 0$
b	Parameter	Interaction rate between prey and predator affecting predator growth	$b \geq 0$
m	Parameter	Natural death rate of predators	$m \geq 0$
c	Parameter	Saturation level of predation	$c > 0$
e	Parameter	Anti-predator behavior level	$e \geq 0$

To simplify the model (5), a transformation will be made by supposing: $x = Ku$, $y = Kv$, and $t = \frac{\tau}{r}$ so as to obtain

$$\begin{aligned} \frac{du}{d\tau} &= u(1-u) - \frac{aKuv}{r(c+Ku)} \\ \frac{dv}{d\tau} &= \frac{bKuv}{r(c+Ku)} - \frac{mv}{r} - \frac{eKuv}{r}. \end{aligned} \tag{6}$$

Suppose that

$$x = u, \quad y = v, \quad t = \tau, \quad \alpha = \frac{a}{r}, \quad \beta = \frac{b}{r}, \quad \delta = \frac{m}{r}, \quad \gamma = \frac{eK}{r}, \quad c = K\eta.$$

Then the model becomes

$$\begin{aligned} \frac{dx}{dt} &= x(1-x) - \frac{\alpha xy}{\eta + x} \\ \frac{dy}{dt} &= \frac{\beta xy}{\eta + x} - \delta y - \gamma xy. \end{aligned} \tag{7}$$

Then, model (7) is used to identify the possible bifurcations that will occur through certain parameter variations.

3.2. Determining the Equilibrium Points of the Model

Determining the equilibrium points $\dot{x}(t) = 0$ and $\dot{y}(t) = 0$. from $\dot{x}(t) = 0$, $x(1-x) - \frac{\alpha xy}{\eta + x} = 0$, it is obtained: $y = \frac{(1-x)(\eta + x)}{\alpha}$, from $\dot{y}(t) = 0$, then $\frac{\beta xy}{\eta + x} - \delta y - \gamma xy = 0$ is obtained: $y = 0$ or $\frac{\beta xy}{\eta + x} - \delta - \gamma x = 0$.

If $y = 0$, then $x(1-x) = 0$ is obtained $x = 0$ or $x = 1$.

If $\frac{\beta x}{\eta + x} - \delta - \gamma x = 0$, then

$$x_{1,2} = \frac{\beta - \delta - \gamma\eta \pm \sqrt{(-\beta + \delta + \gamma\eta)^2 - 4\gamma\delta\eta}}{2\gamma}.$$

For

$$x = \frac{\beta - \delta - \gamma\eta - \sqrt{(-\beta + \delta + \gamma\eta)^2 - 4\gamma\delta\eta}}{2\gamma},$$

it is obtained

$$y = \frac{\left(1 - \frac{\beta-\delta-\gamma\eta-\sqrt{(-\beta+\delta+\gamma\eta)^2-4\gamma\delta\eta}}{2\gamma}\right)\left(\eta + \frac{\beta-\delta-\gamma\eta-\sqrt{(-\beta+\delta+\gamma\eta)^2-4\gamma\delta\eta}}{2\gamma}\right)}{\alpha}.$$

For

$$x = \frac{\beta - \delta - \gamma\eta + \sqrt{(-\beta + \delta + \gamma\eta)^2 - 4\gamma\delta\eta}}{2\gamma},$$

it is obtained

$$y = \frac{\left(1 - \frac{\beta-\delta-\gamma\eta+\sqrt{(-\beta+\delta+\gamma\eta)^2-4\gamma\delta\eta}}{2\gamma}\right)\left(\eta + \frac{\beta-\delta-\gamma\eta+\sqrt{(-\beta+\delta+\gamma\eta)^2-4\gamma\delta\eta}}{2\gamma}\right)}{\alpha}.$$

Suppose that

$$X_1 = \frac{\beta - \delta - \gamma\eta - \sqrt{(-\beta + \delta + \gamma\eta)^2 - 4\gamma\delta\eta}}{2\gamma}, \quad Y_1 = \frac{(1 - X_1)(\eta + X_1)}{\alpha}.$$

$$X_2 = \frac{\beta - \delta - \gamma\eta + \sqrt{(-\beta + \delta + \gamma\eta)^2 - 4\gamma\delta\eta}}{2\gamma}, \quad Y_2 = \frac{(1 - X_2)(\eta + X_2)}{\alpha}.$$

So that four equilibrium points are obtained, namely

$$E_1 = (0, 0), \quad E_2 = (1, 0), \quad E_3 = (X_1, Y_1), \quad \text{and} \quad E_4 = (X_2, Y_2).$$

3.3. Stability Analysis of the Equilibrium Point of the Model

Determine the Jacobian matrix of the model

$$D_f = \begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{pmatrix} = \begin{pmatrix} 1 - 2x + \frac{\alpha xy}{(\eta + x)^2} - \frac{\alpha y}{\eta + x} & -\frac{\alpha x}{\eta + x} \\ -\frac{\beta xy}{(\eta + x)^2} + \frac{\beta y}{\eta + x} - \gamma y & \frac{\beta x}{\eta + x} - \gamma x - \delta \end{pmatrix}.$$

Stability of Equilibrium Points E_1 and E_2.

1. The equilibrium point E_1 is always unstable and of saddle type. Because, if the equilibrium point E_1 is substituted into the Jacobian matrix of the model, we obtain the eigenvalues $\lambda = 1$ and $\lambda = -\delta$.
2. The equilibrium point E_2 is unstable and of saddle type if $\frac{\beta}{\eta + 1} - \gamma - \delta > 0$. Because, if the equilibrium point E_2 is substituted into the Jacobian matrix of the model, one eigenvalue is positive and the other one is negative.
3. The equilibrium point E_2 is stable and of sink type if $\frac{\beta}{\eta + 1} - \gamma - \delta < 0$. Because, if the equilibrium point E_2 is substituted into the Jacobian matrix of the model, both eigenvalues are negative.

Stability of Equilibrium Points E_3 and E_4. Suppose $X = X_1$ or X_2, and $Y = Y_1$ or Y_2. If the Jacobian matrix is evaluated at E_3, then use $(X, Y) = (X_1, Y_1)$, and if the Jacobian matrix is evaluated at E_4, then use $(X, Y) = (X_2, Y_2)$.

$$D_{f(E_3)} \text{ or } D_{f(E_4)} = \begin{pmatrix} 1 - 2X + \dfrac{\alpha XY}{(\eta + X)^2} - \dfrac{\alpha Y}{\eta + X} & -\dfrac{\alpha X}{\eta + X} \\ -\dfrac{\beta XY}{(\eta + X)^2} + \dfrac{\beta Y}{\eta + X} - \gamma Y & \dfrac{\beta X}{\eta + X} - \gamma X - \delta \end{pmatrix}.$$

Suppose that

$M_{11} = 1 - 2X + \dfrac{\alpha XY}{(\eta + X)^2} - \dfrac{\alpha Y}{\eta + X}$,

$M_{12} = -\dfrac{\alpha X}{\eta + X}$,

$M_{21} = -\dfrac{\beta XY}{(\eta + X)^2} + \dfrac{\beta Y}{\eta + X} - \gamma Y$,

$M_{22} = \dfrac{\beta X}{\eta + X} - \gamma X - \delta$.

The eigenvalues are obtained

$$\lambda_{1,2} = \frac{M_{11} + M_{22} \pm \sqrt{(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21})}}{2}.$$

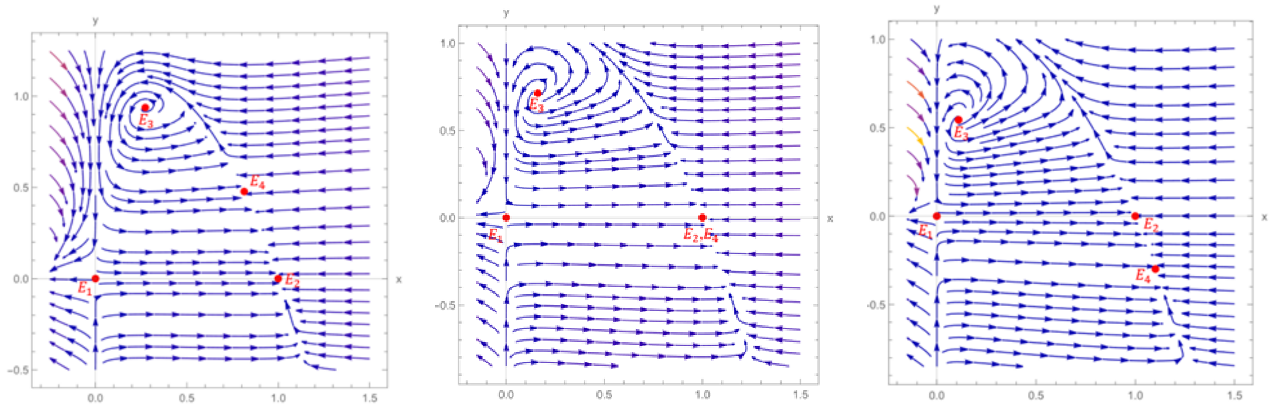
Since all parameters are assumed to be positive, seven cases are obtained, namely

1. If $M_{11} + M_{22} > \sqrt{(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21})} > 0$, the eigenvalues are positive, then the equilibrium points E_3 and E_4 are unstable and of source type.
2. If $M_{11} + M_{22} < \sqrt{(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21})}$, $-(M_{11} + M_{22}) < \sqrt{(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21})}$, and $(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21}) > 0$, then one eigenvalue is positive and the other one is negative, so the equilibrium points E_3 and E_4 are unstable and of saddle type.
3. If $M_{11} + M_{22} < \sqrt{(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21})}$, $-(M_{11} + M_{22}) > \sqrt{(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21})}$, and $(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21}) > 0$, then both eigenvalues are negative, so the equilibrium points E_3 and E_4 are stable and of sink type.
4. If $M_{11} = -M_{22}$ and $(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21}) > 0$, then one eigenvalue is positive and the other is negative, so the equilibrium points E_3 and E_4 are unstable and of saddle type.
5. If $(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21}) < 0$ and $M_{11} > -M_{22}$, a complex eigenvalue with positive real part is obtained, so the equilibrium points E_3 and E_4 are unstable and of spiral source type.
6. If $(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21}) < 0$ and $M_{11} < -M_{22}$, a complex eigenvalue with negative real part is obtained, so the equilibrium points E_3 and E_4 are stable and of spiral sink type.
7. If $(-M_{11} - M_{22})^2 - 4(M_{11}M_{22} - M_{12}M_{21}) < 0$ and $M_{11} = -M_{22}$, the eigenvalues are purely imaginary, then the equilibrium points E_3 and E_4 are stable and of center type.

3.4. bifurcation analysis of the model

Model (7) is used to analyze the bifurcation by setting certain parameter values based on the assumptions used.The parameter values used in this study are selected to ensure biological feasibility (i.e., positive and realistic values), to yield valid equilibrium points, and to capture changes in system stability. Furthermore, the selection of parameter values is intended to clearly demonstrate bifurcation phenomena, which are identified through numerical simulations and exploratory analysis of the model. After going through the analysis process, it is found that the system experiences transcritical, Hopf, and saddle-node bifurcations. The following are the various results of the analysis:

1. Transcritical Bifurcation
   The parameter assumptions before bifurcation occur are $\alpha = 0.425$, $\eta = 0.275$, $\beta = 0.336$, $\delta = 0.125$, and $\gamma = 0.155$. Based on the parameter assumptions, the equilibrium points are obtained, namely: $E_1 = (0, 0)$, $E_2 = (1, 0)$, $E_3 = (0.27253, 0.937204)$, and $E_4 = (0.81376, 0.477107)$. The eigenvalues of E_2 and E_4 are $\lambda_1 = -1$ or $\lambda_2 = -0.01647$ and $\lambda_1 = -0.691449$ or $\lambda_2 = 0.0168885$. By changing the parameter η from 0.275 to 0.2 will result in changes in the behavior of the system. From the change in parameters, the equilibrium points are obtained, namely: $E_1 = (0, 0)$, $E_2 = (1, 0)$, $E_3 = (0.16129, 0.712983)$, and $E_4 = (1, 0)$. The eigenvalues of E_2 and E_4 are the same, i.e. $\lambda_1 = -1$ or $\lambda_2 = 0$. By changing the parameter η from 0.2 to 0.15, the behavior of the system will change. From the change in parameters, the equilibrium points are obtained, namely: $E_1 = (0, 0)$, $E_2 = (1, 0)$, $E_3 = (0.109824, 0.54421)$, and $E_4 = (1.10147, -0.298779)$. The eigenvalues of E_2 and E_4 are $\lambda_1 = -1$ or $\lambda_2 = 0.0121739$ and $\lambda_1 = -1.17914$ or $\lambda_2 = -0.0116416$.



(a) Numerical Simulation of Model (7) Before Transcritical Bifurcation (when parameter $\eta = 0.275$)

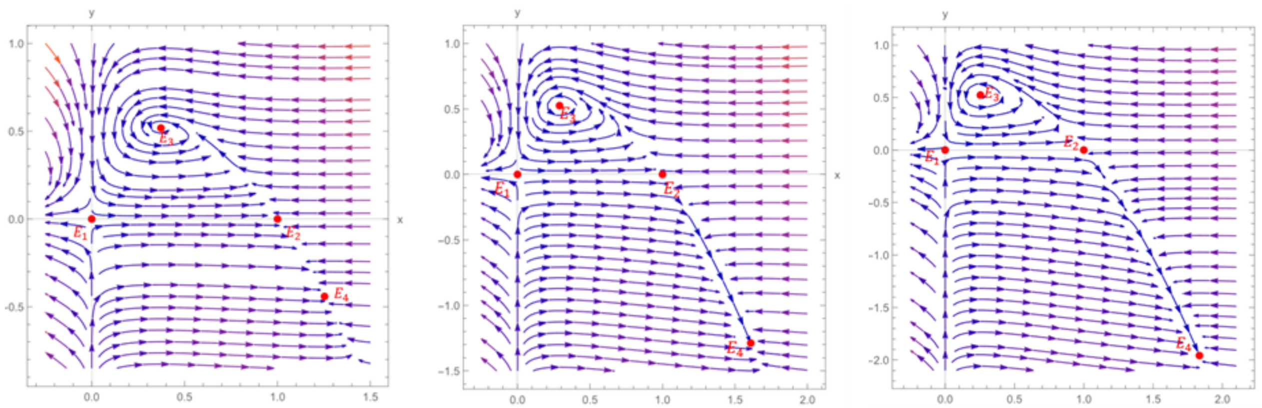
(b) Numerical Simulation of Model (7) at Transcritical Bifurcation (when parameter $\eta = 0.2$)

(c) Numerical Simulation of Model (7) after Transcritical Bifurcation (When parameter $\eta = 0.15$)

Figure 1. Comparison of Numerical Simulations Before, At, and After Transcritical Bifurcation

   Figure 1 shows the occurrence of a transcritical bifurcation, where before the bifurcation, the predator is unable to survive, causing the prey to grow independently since the equilibrium point E_2 is stable. After the bifurcation, the predator can survive together with the prey in a state of coexistence because the stability of the equilibrium point E_2 changes to become unstable.
2. Hopf Bifurcation
   The parameter assumptions before bifurcation occur are $\alpha = 0.96$, $\eta = 0.42$, $\beta = 0.75$, $\delta = 0.264$, and $\gamma = 0.2376$. Based on the parameter assumptions, the equilibrium points are obtained, namely: $E_1 = (0, 0)$, $E_2 = (1, 0)$, $E_3 = (0.372433, 0.518026)$, and $E_4 = (1.25302, -0.440948)$. The eigenvalue of E_3 is $\lambda_1 = -0.0387425 + 0.245378i$ or $\lambda_2 = -0.0387425 - 0.245378i$. Because the eigenvalue of the real part is negative, the stability of the equilibrium point is unstable and spiral sink type. By changing the parameter β from 0.75 to 0.8150405 makes the point E_3 experience a change in behavior. From the change in parameters, the equilibrium points are obtained, namely: $E_1 = (0, 0)$, $E_2 = (1, 0)$, $E_3 = (0.29, 0.525104)$, and $E_4 = (1.60919, -1.28768)$. The eigenvalue of E_3 is $\lambda_1 = -2.47967 \times 10^{-10} - 0.301493i$ or $\lambda_2 = -2.47967 \times 10^{-10} + 0.301493i$. Because the complex eigenvalue and real part are close to zero, the stability of the equilibrium point is stable and center type. By changing the parameter β from 0.8150405 to 0.86, the equilibrium point E_3 changes its behavior. From the change in parameters, the equilibrium points are obtained, namely: $E_1 = (0, 0)$, $E_2 = (1, 0)$, $E_3 = (0.25446, 0.523788)$, and $E_4 = (1.83396, -1.95803)$. The

eigenvalue of E_3 is $\lambda_1 = 0.006844 + 0.177719i$ or $\lambda_2 = 0.006844 - 0.177719i$. Because there is a positive real eigenvalue, the stability of the equilibrium point is unstable and spiral source type.



(a) Numerical Simulation of Model (7) Before Hopf Bifurcation (when parameter $\beta = 0.75$)

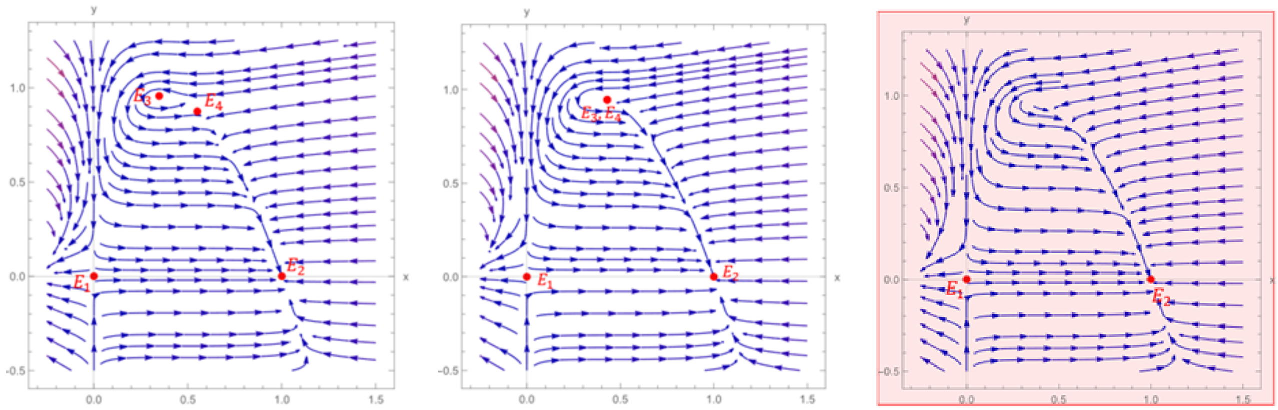
(b) Numerical Simulation of Model (7) at Hopf Bifurcation (when parameter $\beta = 0.8150405$)

(c) Numerical Simulation of Model (7) after Hopf Bifurcation (When parameter $\beta = 0.86$)

Figure 2. Comparison of Numerical Simulations Before, At, and After Hopf Bifurcation

Figure 3 shows the occurrence of a Hopf bifurcation, where before the bifurcation, the predator and prey populations are in a stable equilibrium since the equilibrium point E_3 is stable. At the point of bifurcation, both populations begin to exhibit continuous oscillations because the equilibrium point E_3 becomes a center. Biologically, this means that when predators consume prey, the prey population decreases, causing a shortage of food for predators and consequently a decline in the predator population. As the predator population decreases, the prey population begins to recover. As the prey population increases, the predator population also increases. However, as the predator population rises again, the prey population decreases due to predation. This cycle continues indefinitely, resulting in oscillatory dynamics in both populations.

3. Saddle-Node Bifurcation
The parameter assumptions before bifurcation occur are $\alpha = 0.425$, $\eta = 0.275$, $\beta = 0.336$, $\delta = 0.125$, and $\gamma = 0.18$. Based on the parameter assumptions, the equilibrium points are obtained, namely: $E_1 = (0, 0)$, $E_2 = (1, 0)$, $E_3 = (0.3472, 0.9557)$, and $E_4 = (0.55, 0.873529)$. By changing the parameter γ from 0.18 to 0.18589805 results in changes in the behavior of the system. From the change in parameters, the equilibrium points are obtained, namely: $E_1 = (0, 0)$, $E_2 = (1, 0)$, $E_3 = (0.43, 0.9455)$, and $E_4 = (0.43, 0.9455)$. By changing the parameter γ from 0.18589805 to 0.19 results in changes in the behavior of the system. From the change in parameters, the equilibrium points are obtained, namely: $E_1 = (0, 0)$, $E_2 = (1, 0)$, $E_3 = (0.417763 - 0.08i, 0.964111 + 0.207968i)$, and $E_4 = (0.417763 + 0.08i, 0.964111 - 0.207968i)$. If there is an imaginary equilibrium point, it will not appear in the phase portrait of the system in the state space represented in real terms, so the equilibria E_3 and E_4 are assumed not to exist.



(a) Numerical Simulation of Model (7) Before Saddle-Node Bifurcation (when parameter $\gamma = 0.18$)

(b) Numerical Simulation of Model (7) at Saddle-Node Bifurcation (when parameter $\gamma = 0.18589805$)

(c) Numerical Simulation of Model (7) after Saddle-Node Bifurcation (When parameter $\gamma = 0.19$)

Figure 3. Comparison of Numerical Simulations Before, At, and After Saddle-Node Bifurcation

Figure 3 shows the occurrence of a saddle-node bifurcation, where before the bifurcation, there are two possibilities: the predator population can survive with the prey population in equilibrium, or the predator population goes extinct and the prey evolves alone. After the bifurcation, only one possibility remains: the predator population does not survive, and the prey population evolves on its own. This bifurcation occurs when the parameter γ is changed to a certain value.

The results of this study have important implications for understanding ecological dynamics, particularly in predator–prey interactions involving saturation effects and anti-predator behavior. The findings indicate that small changes in key parameters can significantly alter the stability of the system, leading to different ecological outcomes such as species coexistence, oscillatory dynamics, or extinction of predators. This suggests that environmental factors influencing predation efficiency, prey defense mechanisms, and predator mortality play a crucial role in maintaining ecosystem balance. From a practical perspective, this model can be used as a reference in ecosystem management and conservation strategies. For instance, controlling factors related to prey defense or predator efficiency may help prevent species extinction or uncontrolled population growth. Additionally, the identification of bifurcation points provides insight into critical thresholds where the system undergoes qualitative changes, which is essential for anticipating sudden shifts in population dynamics. Therefore, this study contributes not only to theoretical developments in mathematical biology but also to practical decision-making in ecological management.

4. Conclusions

This study shows that population dynamics in predator-prey systems are very sensitive to changes in parameter values. Small changes in parameter values can cause large changes to the stability of the system, which is shown through the appearance of bifurcations. This result confirms that the system has a high degree of sensitivity to variations in the parameters used in the model. The predator-prey model under study experiences three main types of bifurcations, namely transcritical, Hopf, and saddle-node, each occurring due to changing the parameters η, β, and γ. Future research can extend this study by incorporating more complex biological factors into the model, such as time delays, spatial diffusion, or stochastic effects, to better represent real ecological systems. Additionally, the model can be expanded to include multiple predator or prey species, as well as harvesting or environmental fluctuations, to analyze more realistic ecological interactions.

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